POSTCRITICAL ELASTIC STATES OF STRICTLY CONVEX SHELLS IN A TEMPERATURE FIELD

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Abstract

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In the present note we consider the question of the nature of postcritical elastic states of a shell under the action of a prescribed load in the presence of a heat flux in the body of the shell. By postcritical we mean elastic states characterized by significant changes in the initial form. Usually such states arise as a result of loss of stability of the shell. The heat flux in the body of the shell is assumed to be normal to the middle surface, with isothermal equidistant layers. The temperature distribution through the thickness of the shell is arbitrary.

It is assumed that the fixing of the edge of the shell does not exclude the possibility of small displacements of points of the edge in the tangent planes. Owing to this, the stressed state in an arbitrary section of the shell before loading is characterized by a constant bending moment depending only on the law of temperature distribution through the thickness of the shell. As for the mean values of the stresses in each such section, they are equal to zero.

The problem that we have in mind is to compare the elastic states of two identical shells under the same loading, one of which is in a heat flux with the properties indicated, while the other is not subjected to any thermal action. In particular, we are interested in the influence of the heat flux on the critical elastic states and on the corresponding critical loads.

The postcritical elastic states of a shell in the absence of a heat flux are determined by variational principle A (([1]), Chap. II). According to this principle, the determination of postcritical elastic states of a geometrically inextensible shell under the action of a prescribed load q reduces to a variational problem for a certain functional $W = U(\tilde{F}) - A_q(\tilde{F})$, defined on isometric transformations $\tilde{F}$ with a loss of smoothness and the formation of edges along individual curves γ. The term $U(\tilde{F})$ of this functional is the deformation energy of the shell, and $A_q(\tilde{F})$ is the work done by the load q.

In the case of the presence of a heat flux in the body of the shell, the determination of elastic states can be based on the same variational principle. However, in view of the presence of an initial stressed state of the shell caused by nonuniform heating, the expression for the deformation energy of the shell $U(\tilde{F})$ will be different. Fortunately, the derivation of the corresponding formula for the deformation energy of a shell in a heat flux reduces to the already solved problem for a shell outside a heat flux, by the inclusion of additional terms that do not depend on the varied functions of the solution considered. We shall present this derivation.

As shown in ([1], Chap. II), the deformation energy $U(\tilde{F}) = U_\gamma + U(G)$. The term $U(G)$ is the bending energy of the shell over the basic surface outside a small neighborhood of the edges γ of the isometric transformation $\tilde{F}$. It is determined in the usual way by the formula

$$U(G) = \frac{D}{2} \iint_{\tilde{F}} \left(\Delta k_1^2 + \Delta k_2^2 + 2\nu \Delta k_1 \Delta k_2\right) d\sigma, \tag{*}$$

where D is the bending stiffness of the shell; Δk_1 and Δk_2 are the principal changes in the normal curvatures of the middle surface under its deformation from the initial form F into $\widetilde{F}$, and the integration is over the area of the surface $\widetilde{F}$ with a cut along the edge γ.

The term U_γ is much more complicated; it takes into account the bending energy referred to the edge γ of the isometric transformation $\widetilde{F}$, and the energy of the stretching–compression of the middle surface accompanying this bending. The energy U_γ per unit length of the edge γ is obtained by solving a complicated variational problem for a functional depending on two varied functions $u(s)$ and $v(s)$, which characterize the deformation straightening the edge γ. For brevity of exposition we shall not give the complete expression for this functional and shall confine ourselves to studying those additional terms that are due to temperature stresses.

Since the action of the heat flux is expressed integrally (over the shell thickness) only in the appearance of a constant bending moment m, wherever we compute the bending energy (by formula $(*)$), the normal curvature of the initial surface of the shell must be changed by a certain constant ϑ, equal to the change in curvature under the action of the bending moment m. The quantity ϑ is the change in curvature of a thin strip cut from the shell under the action of the temperature stresses caused by the heat flux.

The indicated change in the normal curvature of the initial surface of the shell in the formula for the deformation energy per unit length of the edge γ leads to the following simple result:

$$\overline{U}_\gamma = \overline{U}_{\gamma|0} - 2D(1+\nu)\vartheta\alpha. \tag{**}$$

Here $\overline{U}_{\gamma|0}$ is the deformation energy in the absence of heat flux, and $\overline{U}_\gamma$ is the energy of the same deformation in the presence of heat flux. The other quantities have their former meanings: D is the bending stiffness; ν is Poisson' s ratio; 2α is the angle between the tangent planes of the surface F along the edge γ.

In formula $(**)$ it is essential to note that the additional term caused by the heat flux in the body of the shell does not contain the functions u and v, which characterize the deformation straightening the edge (they enter into $\overline{U}_{\gamma|0}$). Therefore the functions u and v realizing the minimum of the functional determining the energy U_γ of the true deformation do not depend on the heat flux. As a result, for the deformation energy U_γ straightening the edge, one obtains the expression

$$U_\gamma = U_{\gamma|0} - 2D(1+\nu)\vartheta \int_\gamma \alpha\, ds,$$

where the integration is carried out along the arc of the edge γ.

The term $U(G)$ is corrected analogously in the presence of heat flux. In this case the following expression is obtained:

$$U(G) = U(G)_{|0} + D(1+\nu)\vartheta \iint_{\widetilde{F}} (\Delta k_1 + \Delta k_2)\, d\sigma + U_0,$$

where U_0 is the energy of the initial stressed state of the shell caused by temperature stresses.

Now, when the expression for the deformation energy $U(\widetilde{F})$ in the presence of heat flux has been found, the supercritical elastic states of the shell under the action of a given load are determined by the variational principle A, taking into account the indicated changes in $U(\widetilde{F})$. As shown in (([1]), Ch. III), the study of the supercritical elastic states of shallow strictly convex shells reduces to the consideration of the functional W on the simplest isometric transformations with mirror bulging of the basic form.

Simple calculations show that in this case the terms

$$-2D(1+\nu)\vartheta \int_\gamma \alpha\, ds, \qquad D(1+\nu)\vartheta \iint_{\tilde{F}} (\Delta k_1 + \Delta k_2)\, d\sigma$$

mutually compensate each other, and for the deformation energy of the shell one obtains the following expression

$$U(F) = U(F)\big|_0 + U_0.$$

The term U_0 does not depend on the varied form F. Therefore the variational problem for the functionals W and $W\big|_0$ has one and the same solution F_q. The preceding consideration of the question may be summarized in the following conclusion.

A heat flux in the body of the shell, normal to the middle surface, with isothermal equidistant layers, does not affect the character of supercritical elastic states, in particular the value of the lower critical load. The stressed state in the material of the shell is obtained by superposing the stressed state in the absence of heat flux and the stressed state of an unloaded shell in the presence of heat flux.

An analogous consideration may be carried out in the initial stage of supercritical deformation, when the shell loses stability and begins to buckle. It turns out that at this stage as well the heat flux does not affect the character of buckling, and in particular does not change the critical load.

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1. A. V. Pogorelov, *Geometric Methods in the Nonlinear Theory of Shells*, "Nauka," 1967.

Note: Figure translations are in progress. See original paper for figures.