RECONSTRUCTION OF THE RELIEF OF THE SEA FLOOR IN AREAS NOT COVERED BY SOUNDINGS BY MEANS OF THE METHOD OF LEAST SQUARES

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RECONSTRUCTION OF THE RELIEF OF THE SEA FLOOR IN AREAS NOT COVERED BY SOUNDINGS BY MEANS OF THE METHOD OF LEAST SQUARES

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1. The problem of reconstructing as accurately as possible the relief of the sea floor in areas not covered by soundings, on the basis of depth-measurement data in neighboring regions, may be regarded as a particular case of the classical Gauss problem—the determination of the most reliable values of a function on the basis of experimental data. As is known, the method of least squares was proposed for solving this problem (see, for example, ([1])). Problems of this kind have repeatedly been solved in mathematical statistics, meteorology, and other areas of applied mathematics. However, in constructing bathymetric and navigation charts this method, so far as is known, has not yet been used.

2. In solving problems of this kind, either the trivial method of linear interpolation was applied, or else the method of geomorphological interpolation, which makes it possible to use the experimental information more fully. It is obvious, however, that in order to make full use of the data obtained on the bottom relief, depths at points not covered by soundings should be reconstructed by the method of least squares.

3. Thus, it is required to indicate the most reliable values of the depth $H(x, y)$ at any given point of the sea floor, if the values $H(x_i, y_i)$ are known for a discrete set of sounded points. In processing the experimental data it was assumed that we have the so-called situation of equal-accuracy observations, i.e., that the root-mean-square error of measurement is the same at all points.

4. In determining $H(x, y)$, it is necessary to choose a class of approximating functions. In this work it is assumed that the depth at a given point is approximated by a polynomial of the third degree

$$H(x,y) = \sum_{\substack{ij \\ 0\leq i+j\leq 3}} a_{ij}x^i y^j. \tag{1}$$

The coefficients a_{ij} at a given point (x,y) were determined from the condition of minimizing the expression

$$S(0,0) = \sum_{n=1}^{N} p_n \left[\sum_{\substack{ij \\ 0\leq i+j\leq 3}} a_{ij}x^i y^j - H_n \right]^2. \tag{2}$$

Here N is the number of points covered by soundings; x^i, y^j, H_n are the known coordinates of the points and the depth at them, respectively; p_n is a certain weight factor, taken in the form ([2], [3])

$$p_n = \frac{1}{1+\alpha\left(x_n^2+y_n^2\right)^2}, \tag{3}$$

where α is an empirically selected coefficient, the range of variation of which, as was established as a result of empirical trials, can be

vary within the interval 0.04–0.1 without any appreciable influence on the final result. It is expedient to introduce p_n when approximating by polynomials, since for large values of the arguments the polynomial has, in practice, large absolute values, and because of this the distant measured points enter the computations with an unjustifiably increased weight. Physically, the introduction of the weight factor p_n is adequate to the assumption that, with increasing distance from the point under study, which we take as the origin of coordinates, the mean-square measurement error increases monotonically.

5. In determining N—the number of points with measured depth participating in the minimization of S—we, by empirical trials, settled on a region including from 40 to 90 points. In the case where the point with the sought depth is far from the boundaries of the area, the region was a square centered at this point. For points near the boundaries of the area, the region became a rectangle.

6. The computations were carried out on the "Minsk-2" computer as follows. The depth values with the corresponding coordinates were entered into the machine, after which a selection was made of the points lying inside the specified square centered at the point under investigation. Then the system of normal equations for the coefficients a_{ij} was solved:

$$\sum_{n=1}^{N} x_n^i y_n^j p_n \left[\sum_{\substack{ij \\ 0<i+j<3}} \left(a_{ij} x_n^i y_n^j - H_n \right) \right] = 0. \qquad (4)$$

In (4), N is the number of points with known depths that have fallen inside the square. The coefficients a_{ij} determine the approximating surface; the coefficient a_{00} is the sought depth.

7. For carrying out the calculations, a polygon of approximately 30×8 miles was taken, located in the central part of the Arabian–Indian Ridge. The sounding of the polygon was performed on the research vessel *Vityaz* in 1964. The bottom relief in the selected area is highly dissected; slope angles reach 45°. We note that, in the relief of the ocean floor, angles up to 10° are the most widespread. Thus, the selected sector may be classed among the most complex in terms of bottom relief.

In the area there are 8 sounding tacks of sublatitudinal extent and 3 small tacks crossing the area at an angle close to 45° (Fig. 1a). For processing, depths taken from profiles at distances between them close to 6 cable lengths were used. Profiles Nos. 5, 11, 12, and 14 were not included in the processing. It was required to restore these profiles. Restoration of profile No. 5 was also carried out with a threefold thinning of the measured points, i.e., the distances between them along the profiles were close to 18 cable lengths. The results of the experiment are presented in Fig. 1b, c.

8. As can be seen, the restored profiles agree well with the experimental ones.

The computed profile No. 5 conveys all the main regularities of the relief reflected on the experimental profile. The maximum difference in depth at the extreme points does not exceed 350 m with a depth change along the profile of 2.5 km. However, even this discrepancy can most likely be explained by errors in the coordination of the sounding tacks, as a result of which they turned out to be somewhat displaced relative to one another. A threefold thinning of the points participating in the determination of the depths of profile No. 5 did not, as is evident from Fig. 1c, lead to a substantial change in the restored profile. Comparison of profile No. 5 with the neighboring ones (Nos. 4 and 6) shows (Fig. 1b) that, despite their external similarity, they all differ significantly. It is therefore clear that neither linear interpolation nor geomorphological analysis makes it possible to reliably restore

profile No. 5 from the data of adjacent profiles, whereas the method being applied gives a good result.

The distance between profiles No. 11 and No. 12 is about 1 cable, and therefore they are very similar to one another. The reconstructed profiles are very close to the experimental ones; the discrepancy in depths does not exceed 75 m. If one also takes into account that the tacks passed along a steep slope and that the

Fig. 1

Figure 1: Fig. 1

recording on the echograms is strongly blurred (up to 150 m), then the agreement between the experimental and reconstructed profiles may be considered complete.

Fig. 1. **a** –scheme of the measured tacks on the polygon: **1** –tacks included in the processing; **2** –tacks not included in the processing. **b** –superposed profiles No. 4, No. 5, and No. 6. **c** –profile No. 5: **1** –actual; **2** –reconstructed; **3** –reconstructed using a measuring-data grid thinned by a factor of 3. **d** –profile No. 14: **1** –actual; **2** –reconstructed. Ratio of the vertical and horizontal scales on the profiles: 1 : 37.

An interesting result was obtained in reconstructing profile No. 14 (Fig. 1d). It was impossible to determine the narrow and deep depression in the left part of the profile on the basis of a visual analysis of profiles Nos. 7 and 8. However, the method used revealed it completely. The difference between the depths of the upper parts of the experimental and computed-

profiles. This is explained by the difference in depths shown by the echo sounder at the same places on different tacks. In the left part of the profile it reaches 410 m. The reason for such a discrepancy is the large angles of slope of the bottom, at which the echo sounder on differently oriented tacks gives depth values that differ greatly from one another, since in fact in such dissected areas it determines the echo distances to the nearest part of the bottom, and not the depth beneath the vessel' s keel. Thus it is clear that the reconstructed profile No. 14 also gives very good agreement with the experimental one.

9. The computations carried out convincingly show that, with the aid of the least-squares method, it is possible to reconstruct the relief of the sea floor with sufficient accuracy. Further investigations in this direction will require extensive experimentation on an electronic computer, in particular, for example, in selecting the best class of approximating functions. It may be supposed that approximation by trigonometric functions of the form

$$H(x,y) = \sum_{i=1}^{n} A_i \sin B_i x \cdot \sin C_i y. \tag{5}$$

will prove highly productive. In this case the minimization should be carried out with respect to all parameters (A_i, B_i, C_i).

Obviously, the computational technique set forth above should be used for a wide range of elements studied in oceanology and geophysics.

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Note: Figure translations are in progress. See original paper for figures.