LINES OF EQUAL VALUE OF \(\mu\) IN THE INSTABILITY REGIONS FOR THE MATHIEU EQUATION

MATHEMATICS

1968

SovietRxiv

Fig. 1. Lines of equal value of the characteristic exponent μ

Figure 1: Fig. 1. Lines of equal value of the characteristic exponent μ

Abstract

Full Text

UDC 534.1

MATHEMATICS

Yu. N. Smirnov

LINES OF EQUAL VALUE OF μ IN THE INSTABILITY REGIONS FOR THE MATHIEU EQUATION

(Presented by Academician A. A. Dorodnitsyn, 27 III 1967)

By the characteristic exponent μ for the Mathieu equation

$$d^2y/d\xi^2 + [a - 2q\cos 2\xi]y = 0 \tag{1}$$

is meant the exponent in the general solution of equation (1), written in Floquet form ($\mu \neq 0$)

$$y(\xi) = A_1 e^{\mu\xi} f_1(\xi) + A_2 e^{-\mu\xi} f_2(\xi).$$

Here A_1 and A_2 are arbitrary constants; $f_1(\xi)$ and $f_2(\xi)$ are periodic functions of ξ. The lines of equal value of μ corresponding to the case $\mu = 0$

Fig. 1. Lines of equal value of the characteristic exponent μ

are the lines separating the regions of stability and instability; moreover, in the stability regions μ assumes only imaginary values. The lines of equal value of μ for the regions of stable solutions of the Mathieu equation are well known and, for the first two regions, are given, for example, in the book ([1]).

However, in solving many physical and technical problems ([1,2]), it becomes necessary to know the behavior of the lines of equal value of μ in the instability zones. The nature of these lines is also of independent mathematical interest ([1]). In the literature ([3–5]), up to now only lines of equal value of μ have been known that lie in portions of the instability zones corresponding to small modulation depths $h = q/a \ll 1$; moreover, in the third zone only two lines ([3]) are known, corresponding to the values $\mu = 0.05$ and $\mu = 0.10$. For the indicated modulation depths, the search for the lines is carried out with the aid of Hill' s

Fig. 2. Values of $\mu_{\max}$ as a function of the number n of the instability zone

Figure 2: Fig. 2. Values of $\mu_{\max}$ as a function of the number n of the instability zone

determinant. For values $h > 1$ the problem becomes sharply more complicated mathematically and can be solved only by numerical methods. In the present work, with the aid of an electronic computer, lines of equal value of μ have been found for the first three instability zones of the solutions of Mathieu' s equation (1). The calculation was carried out by the method set forth in ([6]). The results of the computations are presented in Fig. 1.

Fig. 2. Values of $\mu_{\max}$ as a function of the number of the instability zone n

At small modulation depths ($h \ll 1$), as is known, the width of the parametric resonance decreases with increasing number n of the instability zone as h^n, and correspondingly the values of the characteristic exponent μ also decrease from zone to zone (for $h = \text{const}$). At large modulation depths ($h \gtrsim 1$) the picture changes: with increasing number of the instability zone, at one and the same value of h, the maximum value of the characteristic exponent $\mu_{\max}$ for the zone n is less than the maximum value of the characteristic exponent $\mu_{\max}$ for the zone $n+1$. This tendency could have been expected by analyzing the results given in ([3,4]). From Fig. 1 the noted feature follows with obviousness. This means that, at large modulation depths, the resonances corresponding to large values of n become the principal ones, and nonlinearity in a real physical system approximately described by equation (1) will manifest itself the sooner, the higher the order of the parametric resonance realized in it.

It is curious that, if one computes $\mu_{\max}$ for different zones at a constant value of h, then in the plane $(\mu_{\max}, n)$ (Fig. 2), for the given h, they lie practically on straight lines (see also ([7])).

The author expresses sincere gratitude for discussion of the results of the work to D. A. Frank-Kamenetskii, to the participants of V. A. Ditkin' s seminar, and to T. D. Kuznetsova for constant assistance.

Received
16 III 1967

CITED LITERATURE

1. N. W. McLachlan, *Theory and Application of Mathieu Functions*, N. Y., 1964.

2. V. V. Bolotin, *Dynamic Stability of Elastic Systems*, Moscow, 1956.

3. I. I. Goldenblat, A. A. Sizov, *Handbook for the Calculation of Building Structures for Stability and Vibrations*, Moscow-Leningrad, 1952.

4. G. Kotowski, *Zs. angew. Math. u. Mech.*, **23**, 213 (1943).
5. G. Kauderer, *Nonlinear Mechanics*, Moscow, 1961.
6. Yu. N. Smirnov, DAN, **174**, No. 5 (1967).
7. T. D. Kuznetsova, Yu. N. Smirnov, *Tables of Characteristic Exponents for Mathieu' s Equation*, Moscow, 1968.

Note: Figure translations are in progress. See original paper for figures.