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Abstract

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ON THE RESOLVING POWER OF A DIFFRACTION GRATING AND A TELESCOPE

The well-known theory of the resolving power of a diffraction grating and a telescope was given already by Rayleigh ([1]). In brief, the essence of the matter is as follows:

First case. Diffraction grating. Let a plane wave be normally incident on a plane diffraction grating. Let the width of the transparent part of the grating be a, the grating period d, and the total number of grating rulings N; $Nd = A$, where A is the width of the ruled part of the grating. The number of grating rulings per centimeter is, obviously, equal to 1 cm$/d$cm. Let the wavelength be λ. Then at the angle $\arcsin\alpha$, which the normal to the diffracted wave makes with the normal to the grating, there will propagate (as a result of diffraction) a plane wave with intensity

$$I = C_1 \left(\sin\frac{\pi}{\lambda}a\alpha\Big/\frac{\pi}{\lambda}a\alpha\right)^2 \left(\sin N\frac{\pi}{\lambda}d\alpha\Big/\sin\frac{\pi}{\lambda}d\alpha\right)^2 = C_1 y_1 y_2 \qquad (1)$$

(see ([2])). The factor C_1 will play no role in what follows, and it is not necessary to know its exact expression.

The intensity I in expression (1) is the product of two factors y_1 and y_2. Let us dwell on the second:

$$y_2 = \left(\sin N\frac{\pi}{\lambda}d\alpha\Big/\sin\frac{\pi}{\lambda}d\alpha\right)^2. \qquad (2)$$

Let us denote

$$x = \frac{\pi}{\lambda}d\alpha, \qquad (3)$$

then

$$y_2 = (\sin Nx/\sin x)^2. \qquad (4)$$

The numerator of expression (4) is a rapidly oscillating function of x, becoming zero at values $x = k\pi/N$, i.e., at $\alpha = k\lambda/Nd = k\lambda/A$. The extrema of this function lie at $x = (2k+1)\pi/2N$, i.e., $\alpha = (2k+1)\lambda/2Nd = (2k+1)\lambda/2A$.

Table 1

A, cm	0.1	1.0	10
$\Delta\alpha$	$6 \cdot 10^{-4}$	$6 \cdot 10^{-5}$	$6 \cdot 10^{-6}$
$\Delta\xi$, cm	$36 \cdot 10^{-2}$	$36 \cdot 10^{-3}$	$36 \cdot 10^{-4}$

Let us make a numerical example (Table 1). Suppose that $d = 1/6000$ mm and the light from the grating is received by a lens with focal length $F = 6$ m. The waves diffracted by the grating will be collected in the principal focal plane of the lens in the form of many closely spaced points. Let us denote the distance of these points from the optical axis of the lens by ξ; $\xi = F \operatorname{tg} \alpha$. Let us denote by $\Delta\alpha$ the distance between two neighboring extrema (or zeros) of function (4), and by $\Delta\xi$ the distance between their images in the focus of the lens, for which $F = 6$ m.

Let us further investigate the quantity y_2. For small values of x (correspondingly α) one may put $\sin x = x$, and the quantity (4) will take the form

$$y_2 = (\sin Nx/x)^2. \tag{5}$$

Fig. 2. Diffraction pattern near the zero order of a 6-meter concave diffraction grating. The light source is a neon-helium laser. *OO* is the position of the line in the zero order.
a—the line of the neon-helium laser with wavelength 6328 Å in the zero order; slit width 0.01 mm, exposure $2 \cdot 10^{-3}$ s. In all the other photographs the line in the zero order is covered by black paper pressed closely against the photographic film;
b—grating width 150 mm, 600 lines/mm; the vicinity of the zero order is photographed;
c—the grating aperture is stopped down at the sides with black paper, grating aperture 50 mm;
d—grating aperture 10 mm;
e—grating aperture 5 mm. All photographs are enlarged 5 times.

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The extrema of quantity (5) occur at values of Nx close to $(3k+1)\pi/2$, which are given in Table 2 ([3]). For values of x close to $x = (3k+1)\pi/2$, the denominator of expression (4) is close to unity. The quantity y_2 is close to $(\sin Nx)^2$; its extrema are close to $y_2 = 1$. The positions of these extrema are close to the values $x = (3k+1)\pi/2N$. Moreover,

Fig. 1. Graph of the function $y = (\sin 50x/\sin x)^2$. The principal extrema occur at values $x = k\pi$ and are equal to $50^2 = 2500$. The nearest extrema have values of y_2 the same as those given for y_1 in Table 2. Since all subsequent extrema are small, they are shown in the same figure enlarged 2500 times

the quantity y_2 has a large period, equal to $x = k\pi$. The curve connecting the extrema of the function y_2 is shown in Fig. 1. It begins with a large extremum equal to N^2. In the middle part its extrema are close to unity, but are not equal to zero.

Various phenomena arise that are connected with the periodicity of the grating and are grouped under the name of "ghosts." These ghosts become especially noticeable, even in the best gratings, if light from a bright source of monochromatic light, for example a helium-neon laser, falls on the grating ([4], [5]). But when working with a helium-neon laser, even the finest periodic structure of the diffraction image of a bright spectral line, i.e. the factor y_2, sets a limit for observing very weak lines, for example Raman spectra, against the background of these lines (see Fig. 2). In ordinary school manuals this structure, given in Table 2 and in Fig. 2, is considered weak and is neglected. However, it sets a limit to the possibility of detecting weak spectral lines. Usually this structure is perceived as a background of scattered light, but it should be distinguished from such a background. Scattered light, for its part, additionally floods weak spectral lines with light.

Table 2

Extrema of the quantity y_2 for small values of x

	0	4.493	7.725	10.904	14.066
Nx	0	4.493	7.725	10.904	14.066
Intensity	$1N^2$	$0.04718\,N^2$	$0.01649\,N^2$	$0.00834\,N^2$	$0.00503\,N^2$

Second case. Diffraction by the objective of a telescope. But in the case of a telescope as well, the fine diffraction structure

images from a bright object must interfere with the observation of neighboring faint objects. For example, in the case of a circular aperture of radius R, the brightness of the diffraction image from a distant object (from a plane wave) is expressed by the formula

$$I = c\left(2J_1\left(\frac{2\pi}{\lambda}R\alpha\right)\Big/\frac{2\pi}{\lambda}R\alpha\right)^2, \qquad (6)$$

where J_1 is a Bessel function of the first kind with parameter $p = 1$; α is the sine of the angle made by the diffracted ray with the optical axis of the telescope. Denoting

$$z = \frac{2\pi}{\lambda} R\alpha, \tag{7}$$

we write formula (6) in the form

$$I = c(2J_1(z)/z)^2. \tag{8}$$

For large values of z, the function $J_1(z)$ is approximately equal to

$$J_1(z) = \cos(z - 3\pi/4)/\sqrt{\pi z/2}, \tag{9}$$

and the quantity I is expressed by the formula

$$I \approx c \cdot 8\cos^2(z - 3\pi/4)/\pi z^3. \tag{10}$$

Thus, the diffraction pattern of a star in the focus of a telescope has the form of a system of rings with a central spot in the middle, whose brightness at the center may conventionally be taken as unity. The brightness

Table 3

Values of α and z at which a faint luminary can be seen in the vicinity of a bright luminary

n	7	8	9	
$m_2 - m_1$	17.5	20.0	22.5	
$z \cdot 10^{-2} >$	2.44	6.42	13.67	from formula (7)
$\alpha >$	$8''$	$21''$	$45''$	if $2R = 1$ m
$\alpha >$	$1.3''$	$3.5''$	$7.5''$	if $2R = 6$ m

of the distant rings is inversely proportional to the cube of z (the cube of their radius in the principal focal plane of the objective). For a large ring number this decrease is slow, and it must set the limit of visibility of faint luminaries.

Example. At what distance from a bright star can a faint companion still be detected and not be lost in the diffraction halo from the central luminary, if the ratio of their intensities is 10^{-n}, where n takes the values 7, 8, 9, i.e., the difference in stellar magnitudes $m_2 - m_1$ takes the values 17.5, 20.0, 22.5? For this it is necessary that $z^3 > 10^{-n}$, which gives for α and z the values shown in Table 3. For a diffraction grating and a spectroscope, with modern light sources this limit is reached.

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Note: Figure translations are in progress. See original paper for figures.