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Abstract

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MATHEMATICS

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ON COMPLETELY IRREDUCIBLE REPRESENTATIONS OF $SU(2, 1)$

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Let G be a real semisimple Lie group and let $G = UAN$ be its Iwasawa decomposition (see, for example, (7)). Let M be the centralizer of A in G , $MN = \{\mu n, \mu \in M, n \in N\}$. We consider representations of the group G induced by finite-dimensional irreducible representations of MN under which N goes to the identity. The paper describes the structure of these representations for the group $G = SU(2, 1)$ of complex unimodular matrices of order three preserving the indefinite Hermitian form $|z_1|^2 + |z_2|^2 - |z_3|^2$, and finds conditions for equivalence and unitarity of the representations. Representations of this group were considered in (2-5).

1. Let G' be the group, isomorphic to G , of complex unimodular matrices of order three preserving the form $z_1 \bar{z}_3 + z_2 \bar{z}_2 + z_3 \bar{z}_1$. Then

$$G = \left\{ sg' s^{-1} \mid g' \in G', s = s^{-1} = \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{pmatrix} \right\}. \quad (1)$$

Consider the subgroups K, U, Γ, M, N of the group G :

$$K = \{k\} = \left\{ s \begin{pmatrix} \lambda^{-1} e^{i\varphi} & \lambda^{-1} e^{i\varphi} z & \lambda^{-1} e^{i\varphi} (-|z|^2/2 + it) \\ 0 & e^{-2i\varphi} & -e^{2i\varphi} \bar{z} \\ 0 & 0 & \lambda e^{i\varphi} \end{pmatrix} s \right\},$$

$$U = \{u\} = \left\{ \begin{pmatrix} u_{11} & u_{12} & 0 \\ u_{21} & u_{22} & 0 \\ 0 & 0 & u_{33} \end{pmatrix} \right\}, \quad \Gamma = \left\{ \gamma_\varphi : \begin{pmatrix} e^{i\varphi} & 0 & 0 \\ 0 & e^{-2i\varphi} & 0 \\ 0 & 0 & e^{i\varphi} \end{pmatrix} \right\}, \quad (2)$$

$$M = \{\mu \mid \mu \in K, z = 0, t = 0\}, \quad N = \{n \mid n \in K, \lambda = 1, \varphi = 0\},$$

$$K = MN, \quad \Gamma = M \cap N.$$

Here $\lambda > 0$, z is complex, t is real, $u^*u = 1$. Let $g = (a_{jk})_{j,k=1}^3$, $\beta(g) = a_{33} - a_{13}$, $\alpha(g) = |\beta(g)|^{i\sigma - m - 2} \beta(g)^m$ (σ complex, m an integer). Let $\bar{u}\bar{g}$ be an element of U such that $u\bar{g} = k \cdot ug$ for some $k \in K$. Then the representation of the group G , induced by the representation of the subgroup $K = MN: k \rightarrow |k_{33}|^{i\sigma - m} k_{33}^m$, acts in the Hilbert space $\mathcal{L}_2^m(U) = \{f(u) \mid f(u) \in \mathcal{L}_2(U), f(\gamma_\varphi u) = e^{im\varphi} f(u) \text{ for } \gamma_\varphi \in \Gamma\}$ by the formula

$$[T^{\sigma,m}(g)f](u) = \frac{\alpha(ug)}{\alpha(\bar{u}\bar{g})} f(\bar{u}\bar{g}). \quad (3)$$

The operators $T^{\sigma,m}$ are bounded. The representation conjugate to $T^{\sigma,m}$ is $T^{\bar{\sigma},m}$.

Along with representations in $\mathcal{L}_2^m(U)$, it is convenient to consider representations given by formula (3) in the nuclear space $C_m^\infty(U)$ of infinitely differentiable functions on U satisfying the condition $f(\gamma_\varphi u) = e^{im\varphi} f(u)$, $\gamma_\varphi \in \Gamma$.

2. On invariant subspaces of the representations $T^{\sigma,m}$

The representations $\tilde{T}^{\sigma,m}$ in $\mathcal{L}_2^m(U)$ are completely irreducible if: 1) $i\sigma + m$ is not an even number, or 2) $\sigma = m = 0$. If $(i\sigma + m)/2 = p$, where p is an integer, then let us introduce the integer $q = (i\sigma - m)/2$ and denote the representation $T^{(p+q)/i, p-q}$ by $T_{p,q}$.

In the decomposition of the restriction of the representation $T^{\sigma,m}$ to the subgroup U , there enter, with multiplicity one, the irreducible representations of U (isomorphic to $U(2)$) with weights $f_1 = 2\alpha - \beta - m$, $f_2 = \alpha - 2\beta - m$, where $\alpha, \beta \geq 0$ are integers. These representations are realized in subspaces $\mathfrak{M}^{\alpha,\beta}$ spanned by functions from $\mathcal{L}_2^m(\bar{U})$ which are matrix elements of the corresponding irreducible representations: $C_{(\beta-\alpha)/2, j}^{f_1, f_2}$, $|j| \leq (\alpha + \beta)/2$. The invariant subspaces of the representations $T_{p,q}$ are direct sums of the subspaces $\mathfrak{M}^{\alpha,\beta}$. Introduce the following notation:

$$\begin{aligned} \mathfrak{M}_1^{p,q} &= \sum_{\substack{0 \leq \alpha < p \\ 0 \leq \beta < +\infty}} \oplus \mathfrak{M}^{\alpha,\beta}; & \mathfrak{M}_2^{p,q} &= \sum_{\substack{0 \leq \alpha < +\infty \\ 0 \leq \beta < q}} \oplus \mathfrak{M}^{\alpha,\beta}; \\ \mathfrak{M}_{p,q}^1 &= \sum_{\substack{-q < \alpha < +\infty \\ 0 \leq \beta < +\infty}} \oplus \mathfrak{M}^{\alpha,\beta}; & \mathfrak{M}_{p,q}^2 &= \sum_{\substack{0 \leq \alpha < +\infty \\ -p < \beta < +\infty}} \oplus \mathfrak{M}^{\alpha,\beta}. \end{aligned}$$

Of course, these subspaces may turn out to be trivial. The structure of the representation $T_{p,q}$ depends on the signs of the numbers $p, q, p + q$.

- a) $p \geq 1, q \geq 1$. The subspaces $\mathfrak{M}_1^{p,q}; \mathfrak{M}_2^{p,q}; \mathfrak{M}_1^{p,q} + \mathfrak{M}_2^{p,q}; \mathfrak{M}_1^{p,q} \cap \mathfrak{M}_2^{p,q}$ are invariant. The representations induced by $T_{p,q}$ in $\mathfrak{M}_1^{p,q} \cap \mathfrak{M}_2^{p,q}; \mathfrak{M}_1^{p,q} / \mathfrak{M}_1^{p,q} \cap \mathfrak{M}_2^{p,q}; \mathfrak{M}_2^{p,q} / \mathfrak{M}_1^{p,q} \cap \mathfrak{M}_2^{p,q}; \mathcal{L}_2^{p-q}(U) / \mathfrak{M}_1^{p,q} + \mathfrak{M}_2^{p,q}$ are completely irreducible.

- b) $p \leq -1, q \leq -1$. The subspaces $\mathfrak{M}_{p,q}^1; \mathfrak{M}_{p,q}^2; \mathfrak{M}_{p,q}^1 + \mathfrak{M}_{p,q}^2; \mathfrak{M}_{p,q}^1 \cap \mathfrak{M}_{p,q}^2$ are invariant. The representations induced by $T_{p,q}$ in $\mathfrak{M}_{p,q}^1 \cap \mathfrak{M}_{p,q}^2; \mathfrak{M}_{p,q}^1/\mathfrak{M}_{p,q}^1 \cap \mathfrak{M}_{p,q}^2; \mathfrak{M}_{p,q}^2/\mathfrak{M}_{p,q}^1 \cap \mathfrak{M}_{p,q}^2; \mathcal{L}_2^{p-q}(U)/\mathfrak{M}_{p,q}^1 + \mathfrak{M}_{p,q}^2$ are completely irreducible.
- c) $p \geq 1, q \leq -1, p+q \geq 1$. $\mathfrak{M}_1^{p,q}; \mathfrak{M}_{p,q}^1; \mathfrak{M}_1^{p,q} \cap \mathfrak{M}_{p,q}^1$ are invariant. The representations in $\mathfrak{M}_1^{p,q} \cap \mathfrak{M}_{p,q}^1; \mathfrak{M}_1^{p,q}/\mathfrak{M}_1^{p,q} \cap \mathfrak{M}_{p,q}^1; \mathfrak{M}_{p,q}^1/\mathfrak{M}_1^{p,q} \cap \mathfrak{M}_{p,q}^1$ are completely irreducible.
- d) $p \geq 1, q \leq -1, p+q \leq 1$. $\mathfrak{M}_1^{p,q}; \mathfrak{M}_{p,q}^1; \mathfrak{M}_1^{p,q} \oplus \mathfrak{M}_{p,q}^1$ are invariant. The representations in $\mathfrak{M}_1^{p,q}; \mathfrak{M}_{p,q}^1; \mathcal{L}_2^{p-q}(U)/\mathfrak{M}_1^{p,q} \oplus \mathfrak{M}_{p,q}^1$ are completely irreducible.
- e) $p \leq -1, q \geq 1, p+q \geq 1$. $\mathfrak{M}_2^{p,q}; \mathfrak{M}_{p,q}^2; \mathfrak{M}_2^{p,q} \cap \mathfrak{M}_{p,q}^2$ are invariant. The representations in $\mathfrak{M}_2^{p,q} \cap \mathfrak{M}_{p,q}^2; \mathfrak{M}_2^{p,q}/\mathfrak{M}_2^{p,q} \cap \mathfrak{M}_{p,q}^2; \mathfrak{M}_{p,q}^2/\mathfrak{M}_2^{p,q} \cap \mathfrak{M}_{p,q}^2$ are completely irreducible.
- f) $p \leq -1, q \geq 1, p+q \leq -1$. $\mathfrak{M}_2^{p,q}; \mathfrak{M}_{p,q}^2; \mathfrak{M}_2^{p,q} \oplus \mathfrak{M}_{p,q}^2$ are invariant. The representations in $\mathfrak{M}_2^{p,q}; \mathfrak{M}_{p,q}^2; \mathcal{L}_2^{p-q}(U)/\mathfrak{M}_2^{p,q} \oplus \mathfrak{M}_{p,q}^2$ are completely irreducible.
- g) $p = 0, q \geq 1$. $\mathfrak{M}_2^{0,q}$ is invariant. The representations in $\mathfrak{M}_2^{0,q}$ and $\mathcal{L}_2^{-q}(U)/\mathfrak{M}_2^{0,q}$ are completely irreducible.
- h) $p = 0, q \leq -1$. $\mathfrak{M}_{0,q}^1$ is invariant. The representations in $\mathfrak{M}_{0,q}^1$ and $\mathcal{L}_2^{-q}(U)/\mathfrak{M}_{0,q}^1$ are completely irreducible.
- i) $p \geq 1, q = 0$. $\mathfrak{M}_1^{p,0}$ is invariant. The representations in $\mathfrak{M}_1^{p,0}$ and $\mathcal{L}_2^p(U)/\mathfrak{M}_1^{p,0}$ are completely irreducible.
- j) $p \leq -1, q = 0$. $\mathfrak{M}_{p,0}^2$ is invariant. The representations in $\mathfrak{M}_{p,0}^2$ and $\mathcal{L}_2^p(U)/\mathfrak{M}_{p,0}^2$ are completely irreducible.
- k) $p \geq 1, q \leq -1, p+q = 0$. $\mathfrak{M}_1^{p,q}; \mathfrak{M}_{p,q}^1$ are invariant, $\mathfrak{M}_1^{p,q} \oplus \mathfrak{M}_{p,q}^1 = \mathcal{L}_2^{p-q}(U)$. The representations in $\mathfrak{M}_1^{p,q}$ and $\mathfrak{M}_{p,q}^1$ are completely irreducible.
- l) $p \leq -1, q \geq 1, p+q = 0$. $\mathfrak{M}_2^{p,q}; \mathfrak{M}_{p,q}^2$ are invariant, $\mathfrak{M}_2^{p,q} \oplus \mathfrak{M}_{p,q}^2 = \mathcal{L}_2^{p-q}(U)$. The representations in $\mathfrak{M}_2^{p,q}$ and $\mathfrak{M}_{p,q}^2$ are completely irreducible.
3. **On the equivalence* of representations.** If $i\sigma + m$ is not an even number, then the representation $T^{\sigma,m}$ is not equivalent to any of the subrepresentations or factor-representations $T_{p,q}$.

If $i\sigma_1 + m_1$ and $i\sigma_2 + m_2$ are both not even numbers, then the representations T^{σ_1,m_1} and T^{σ_2,m_2} are equivalent if and only if $\sigma_1 = \sigma_2, m_1 = m_2$, or $\sigma_1 = -\sigma_2, m_1 = m_2$.

The representations $T_{p,q}$ are pairwise inequivalent, but the following equivalences occur:

- 1) The subrepresentation $T_{p,q}$ with $p \geq 1$, $q \geq 1$ in the subspace $\mathfrak{M}_1^{p,q} \cap \mathfrak{M}_2^{p,q}$ is equivalent to the factor-representation $T_{-q,-p}$ in $\mathcal{L}_2^{p-q}(u)/\mathfrak{M}_{-q,-p}^1 + \mathfrak{M}_{-q,-p}^2$.
- 2) The subrepresentation $T_{p,q}$ with $p \geq 1$, $q \leq -1$, $p+q \leq -1$ in the subspace $\mathfrak{M}_1^{p,q}$ is equivalent to the factor-representation $T_{-q,-p}$ in $\mathcal{L}_2^{p-q}(u)/\mathfrak{M}_{-q,-p}^1 = \mathfrak{M}_1^{-q,-p}/\mathfrak{M}_1^{-q,-p} \cap \mathfrak{M}_{-q,-p}^1$.
- 3) The subrepresentation $T_{p,q}$ with $p \leq -1$, $q \geq 1$, $p+q \leq -1$ in the subspace $\mathfrak{M}_2^{p,q}$ is equivalent to the factor-representation $T_{-q,-p}$ in $\mathcal{L}_2^{p-q}(u)/\mathfrak{M}_{-q,-p}^2 = \mathfrak{M}_2^{-q,-p}/\mathfrak{M}_2^{-q,-p} \cap \mathfrak{M}_{-q,-p}^2$.
- 4) The subrepresentation $T_{0,q}$ with $q \geq 1$ in $\mathfrak{M}_2^{0,q}$ is equivalent to the factor-representation $T_{-q,0}$ in $\mathcal{L}_2^{-q}(u)/\mathfrak{M}_{-q,0}^2$.
- 5) The subrepresentation $T_{p,0}$ with $p \geq 1$ in $\mathfrak{M}_1^{p,0}$ is equivalent to the factor-representation $T_{0,-p}$ in $\mathcal{L}_2^p(u)/\mathfrak{M}_{0,-p}^1$.
- 6) The factor-representation $T_{p,q}$ with $p \geq 1$, $q \geq 1$ in the factor-space

$$\mathfrak{M}_1^{p,q}/\mathfrak{M}_1^{p,q} \cap \mathfrak{M}_2^{p,q}$$

is equivalent to the factor-representation $T_{-q,-p}$ in

$$\mathfrak{M}_{-q,-p}^2/\mathfrak{M}_{-q,-p}^1 \cap \mathfrak{M}_{-q,-p}^2,$$

to the subrepresentation $T_{p+q,-q}$ in $\mathfrak{M}_{p+q,-q}^1 \cap \mathfrak{M}_{p+q,-q}^1$, and to the factor-representation $T_{q,-p-q}$ in

$$\mathcal{L}_2^{p+2q}(u)/\mathfrak{M}_1^{q,-p-q} \oplus \mathfrak{M}_{q,-p-q}^1.$$

- 7) The factor-representation $T_{p,q}$ with $p \geq 1$, $q \geq 1$ in

$$\mathfrak{M}_2^{p,q}/\mathfrak{M}_1^{p,q} \cap \mathfrak{M}_2^{p,q}$$

is equivalent to the factor-representation $T_{-q,-p}$ in

$$\mathfrak{M}_{-q,-p}^1/\mathfrak{M}_{-q,-p}^1 \cap \mathfrak{M}_{-q,-p}^2,$$

to the subrepresentation $T_{-p,p+q}$ in $\mathfrak{M}_{-p,p+q}^2 \cap \mathfrak{M}_{-p,p+q}^2$, and to the factor-representation $T_{-p-q,p}$ in

$$\mathcal{L}_2^{-2p-q}(u)/\mathfrak{M}_2^{-p-q,p} \oplus \mathfrak{M}_{-p-q,p}^2.$$

- 8) The factor-representation $T_{p,q}$ with $p \geq 1$, $q \geq 1$ in

$$\mathcal{L}_2^{p-q}(u)/\mathfrak{M}_1^{p,q} + \mathfrak{M}_2^{p,q}$$

is equivalent to the subrepresentation $T_{-q,-p}$ in $\mathfrak{M}_{-q,-p}^1 \cap \mathfrak{M}_{-q,-p}^2$, to the subrepresentation $T_{q,-p-q}$ in $\mathfrak{M}_{q,-p-q}^1$, to the subrepresentation $T_{-p-q,p}$ in $\mathfrak{M}_{-p-q,p}^2$, to the factor-representation $T_{p+q,-q}$ in

$$\mathcal{L}_2^{p+2q}(u)/\mathfrak{M}_1^{p+q,-q} = \mathfrak{M}_1^{p+q,-q}/\mathfrak{M}_1^{p+q,-q} \cap \mathfrak{M}_1^{p+q,-q}$$

and to the factor-representation $T_{-p,p+q}$ in

$$\mathcal{L}_2^{-q-2p}(u)/\mathfrak{M}_2^{-p,p+q} = \mathfrak{M}_2^{-p,p+q}/\mathfrak{M}_2^{-p,p+q} \cap \mathfrak{M}_2^{-p,p+q}.$$

- 9) The subrepresentation $T_{0,-q}$ with $q \leq -1$ in $\mathfrak{M}_{0,q}^1$ is equivalent to the factor-representation $T_{-q,0}$ in $\mathcal{L}_2^{-q}(u)/\mathfrak{M}_1^{-q,0}$ and to the subrepresentation $T_{q,-q}$ in $\mathfrak{M}_{q,-q}^2$.
- 10) The subrepresentation $T_{p,0}$ with $p \leq -1$ in $\mathfrak{M}_{p,0}^2$ is equivalent to the factor-representation $T_{0,-p}$ in $\mathcal{L}_2^p(u)/\mathfrak{M}_2^{0,-p}$ and to the subrepresentation $T_{p,-p}$ in $\mathfrak{M}_{-p,p}^1$.

In all other cases the representations are pairwise inequivalent.

Thus, in each set of pairwise equivalent representations there is at least one representation in an irreducible invariant subspace $\mathcal{L}_2^{p-q}(u)$ and at least one representation in the factor-space of the space of the representation $T_{p,q}$ by a maximal invariant subspace.

4. **On the unitarity of the representations $T^{\sigma,m}$ and the subrepresentations $T_{p,q}$.** In $C_m^\infty(u)$ one can introduce a nonzero Hermitian functional invariant with respect to $T^{\sigma,m}$ only for: 1) real σ , 2) purely imaginary σ . In the first case the functional is definite and has the form

$$B(\varphi, \psi) = \int_U \varphi(u) \overline{\psi(u)} du,$$

where du is Haar measure on U . In the second case

* Here and below, in the sense of (1).

the functional is definite for $m = 0$, $-2 < i\sigma < 2$ and $-1 < i\sigma < 1$, $m = 2k + 1$, k an integer. At the other nonintegral points the functional for purely imaginary σ gives an indefinite scalar product with an infinite number of positive and negative squares.

For all subrepresentations $T_{p,q}$ there exist invariant continuous Hermitian functionals. They are definite in the following cases.

The subrepresentation $T_{1,1}$ in the one-dimensional subspace $\mathfrak{M}_1^{1,1} \cap \mathfrak{M}_2^{1,1}$.

The subrepresentations $T_{p,q}$ with $p \geq 1$, $q \leq -1$, $p + q \leq -1$ in the subspace $\mathfrak{M}_1^{p,q}$.

The subrepresentations $T_{p,q}$ with $p \leq -1$, $q \geq 1$, $p + q \leq -1$ in the subspace $\mathfrak{M}_2^{p,q}$.

The subrepresentation $T_{0,1}$ in $\mathfrak{M}_2^{0,1}$.

The subrepresentation $T_{1,0}$ in $\mathfrak{M}_1^{1,0}$.

The subrepresentations $T_{p,-p+1}$ for $p \geq 2$ in $\mathfrak{M}_1^{p,-p+1} \cap \mathfrak{M}_{p,-p+1}^1$.

The subrepresentations $T_{1-q,q}$ for $q \geq 2$ in $\mathfrak{M}_2^{1-q,q} \cap \mathfrak{M}_{1-q,q}^2$.

The subrepresentations $T_{p,q}$ with $p \leq -1$, $q \leq -1$ in $\mathfrak{M}_{p,q}^1 \cap \mathfrak{M}_{p,q}^2$.

The subrepresentations $T_{p,-p}$ for $p \leq -1$ in $\mathfrak{M}_{p,-p}^2$ and $\mathfrak{M}_{p,-p}^2$.

The subrepresentations $T_{p,-p}$ for $p \geq 1$ in $\mathfrak{M}_1^{p,-p}$ and $\mathfrak{M}_{p,-p}^1$.

Invariant Hermitian functionals for representations not equivalent to those listed above are indefinite and, for infinite-dimensional representations, contain an infinite number of both positive and negative squares.

5. Using the result of Harish-Chandra ((6), Theorem 4), we obtain:

Theorem. *Every completely irreducible representation of the group $SU(2,1)$ in a Banach space is equivalent (in the sense of (1)) to one of the representations described in Section 2. Every irreducible unitary representation of $SU(2,1)$ is unitarily equivalent to one of the representations indicated in Section 4.*

6. Unitary representations of the group $SU(2,1)$ were studied in papers (2-5). In (2,4) the representations $T^{\sigma,m}$ are described for real σ and the subrepresentations $T_{p,q}$: for the cases $p \geq 1$, $q \leq -1$, $p + q \leq -1$ and $p \geq 1$, $p + q = 0$ in the subspace $\mathfrak{M}_1^{p,q}$, and for the cases $p \leq -1$, $q \geq 1$, $p + q \leq -1$ and $q \geq 1$, $p + q = 0$ in the subspace $\mathfrak{M}_2^{p,q}$. In (3), moreover, the subrepresentations $T_{p,q}$ with $p \leq -1$, $q \leq -1$ in $\mathfrak{M}_{p,q}^1 \cap \mathfrak{M}_{p,q}^2$, the subrepresentations $T_{p,-p}$ for $p \leq -1$ in $\mathfrak{M}_{p,-p}^2$, and $T_{p,-p}$ for $p \geq 1$ in $\mathfrak{M}_{|p|,-p}^1$ are indicated.

In (5) there are constructed (in infinitesimal form) the subrepresentations $T_{p,q}$ for $p \geq 1$, $q \leq -1$, $p + q \leq -1$ in $\mathfrak{M}_1^{p,q}$, the subrepresentations $T_{p,q}$ for $p \leq -1$, $q \geq 1$, $p + q \leq -1$ in $\mathfrak{M}_2^{p,q}$, and the subrepresentations $T_{p,q}$ for $p \leq -1$, $q \leq -1$ in $\mathfrak{M}_{p,q}^1 \cap \mathfrak{M}_{p,q}^2$. From the continuous series of unitary representations of the Lie algebra, given by the complex number σ (see item 6 of the addendum in (5)), only the representations with integral $\sigma + \bar{\sigma} = m$ extend to representations of the group. These are the representations $T^{\text{Im } \sigma, m}$ (reducible for $\text{Im } \sigma = 0$ and even m).

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