ROTATION OF THE PLANE OF POLARIZATION OF LIGHT CAUSED BY A WEAK INHOMOGENEITY OF THE MEDIUM

PHYSICS

1968

SovietRxiv

Abstract

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UDC 538.3

PHYSICS

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ROTATION OF THE PLANE OF POLARIZATION OF LIGHT CAUSED BY A WEAK INHOMOGENEITY OF THE MEDIUM

(Presented by Academician Ya. B. Zel' dovich on 13 II 1967)

It is known that, when an electromagnetic wave propagates in spatially inhomogeneous nonabsorbing isotropic media, a rotation of the plane of polarization of light occurs ([1]), caused by the twisting of the ray.

In ([1]) it was shown that, from the conditions for the existence of the first approximation with respect to the wavelength λ from Maxwell' s equations, there follows not only the law of variation of the light intensity along the ray, but also the law of rotation of the field vectors **E** and **H**, which has the form:

$$d\varphi/ds = 1/T, \tag{1}$$

where φ is the angle between the vector **E** and the principal normal to the ray $\vec{\tau}$; T is the radius of twisting of the ray; s is the path length measured along the ray.

In ([2]), by S. M. Rytov' s method, in the first approximation a refined law of rotation of the field vectors was obtained,

$$d\varphi/ds = 1/T + {}^1\!/_2\, \vec{\tau} \operatorname{rot} \vec{\tau}. \tag{2}$$

Below it is shown that law (2) is exact and follows without any approximations from Maxwell' s equations. The latter means that higher approximations with respect to the wavelength λ (second, etc.) do not lead to additional contributions to expression (2).

Physically, the rotation of the plane of polarization in the case under consideration is explained as follows ([3]). From Maxwell' s equations

$$\operatorname{rot} \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \qquad \operatorname{div} \mathbf{B} = 0,$$

$$\operatorname{rot}\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}, \qquad \operatorname{div}\mathbf{D} = 0, \tag{3}$$

where $\mathbf{B} = \mu(x, y, z)\mathbf{H}$ and $\mathbf{D} = \varepsilon(x, y, z)\mathbf{E}$, it follows that the vector $\mathbf{E}$ obeys the equation

$$\nabla^2\mathbf{E} + k^2\varepsilon\mu\mathbf{E} = -\operatorname{grad}\frac{(\nabla\varepsilon\,\mathbf{E})}{\varepsilon} - ik[\nabla\mu, \mathbf{H}], \qquad k = \frac{\omega}{c}. \tag{4}$$

In the absence of the right-hand side, i.e., in homogeneous media, the components of the vector $\mathbf{E}$ satisfy identical equations. However, for inhomogeneous media the orientation of $\mathbf{E}$ and $\mathbf{H}$ relative to $\nabla\varepsilon$ and $\nabla\mu$ proves to be essential. In this case the components of the vector $\mathbf{E}$ satisfy different equations, which is what causes the rotation of the plane of polarization when the wave propagates in such a medium.

For a monochromatic plane wave $\sim \exp\{i(\mathbf{kr} - \omega t)\}$, equations (3) can be rewritten in the form

$$\mathbf{H} = a[\vec{\tau}, \mathbf{E}], \qquad \mathbf{E} = -\frac{1}{a}[\vec{\tau}, \mathbf{H}], \tag{5}$$

where $a = \sqrt{\varepsilon/\mu}$, and $\vec{\tau} = c\mathbf{k}/\omega\sqrt{\varepsilon\mu} = \mathbf{n}/\sqrt{\varepsilon\mu}$ is the unit vector along the ray. From equations (5) it is seen that the three vectors $\mathbf{E}$, $\mathbf{H}$, and $\vec{\tau}$ form a natural

···an isosceles trihedron; therefore their general solution may be written in the following form:

$$\mathbf{E} = \frac{1}{a}\left(f_1\mathbf{N} + f_2\mathbf{b}\right), \qquad \mathbf{H} = f_1\mathbf{b} - f_2\mathbf{N}, \tag{6}$$

where $\mathbf{N}$ and $\mathbf{b}$ are, respectively, the normal and binormal to the ray $\vec{\tau}$, while f_1 and f_2 are arbitrary functions of the coordinates. Solving (6) with respect to $\mathbf{N}$ and $\mathbf{b}$, we obtain

$$\mathbf{N} = a\psi_1\mathbf{E} - \psi_2\mathbf{H}, \qquad \mathbf{b} = a\psi_2\mathbf{E} + \psi_1\mathbf{H}, \qquad \psi_{1,2} = f_{1,2}\left(f_1^2 + f_2^2\right)^{-1}.$$

Computing the expressions $\mathbf{N}\operatorname{rot}\mathbf{N}$, $\mathbf{b}\operatorname{rot}\mathbf{b}$, and adding them, we find that

$$\frac{1}{2}\left(\mathbf{N}\operatorname{rot}\mathbf{N} + \mathbf{b}\operatorname{rot}\mathbf{b}\right) = f_1(\vec{\tau}\nabla\psi_2) - f_2(\vec{\tau}\nabla\psi_1) = (\vec{\tau}\nabla)\varphi,$$

where $\varphi = \arctan f_2/f_1$ is the angle between the principal normal to the ray and the vector $\mathbf{E}$.

Noting that $(\vec{\tau}\nabla)\varphi = d\varphi/ds$ and using the identity known from differential geometry,

$$\mathbf{N}\,\mathrm{rot}\,\mathbf{N} + \mathbf{b}\,\mathrm{rot}\,\mathbf{b} = \frac{2}{T} + \vec{\tau}\,\mathrm{rot}\,\vec{\tau},$$

we obtain the simple equation determining the law of rotation of the plane of polarization (2).

Let us consider two examples. As was shown in paper ([2]), the equations of the electromagnetic field in vacuum in the presence of finite masses can be written in the form of Maxwell' s equations (3) for a moving anisotropic medium. In this case the character of the anisotropy is determined by the properties of the metric tensor g_{ik}:

$$\mathbf{D} = \varepsilon\mathbf{E} - [\mathbf{g}, \mathbf{H}], \qquad \mathbf{B} = \mu\mathbf{H} + [\mathbf{g}, \mathbf{E}], \tag{7}$$

where $\varepsilon = (-g)^{1/2}(-g^{00})g^{11}$, $\mu = (-g)^{-1/2}(g^{11})^{-2}$, and the vector $\mathbf{g}$, in the case of a weak gravitational field created by a rotating mass, has the form

$$g_\alpha = g^\alpha = \frac{2k}{c^3 r^3}[\mathbf{r}, \mathbf{M}]_\alpha.$$

Here $\mathbf{M}$ is the angular momentum of the body. As above, from equations (3) and (7) it is easy to obtain that

$$\mathbf{H} = a[\vec{\tau}, \mathbf{E}], \qquad \mathbf{E} = -\frac{1}{a}[\vec{\tau}, \mathbf{H}],$$

where $\vec{\tau} = (\mathbf{n} - \mathbf{g})/\sqrt{\varepsilon\mu}$. Formula (2) gives the law of rotation of the plane of polarization of light as it propagates near a rotating body ([2]). As was noted in paper ([2]), in the case of a spherically symmetric static Schwarzschild field (when $\mathbf{g} = 0$) the ray trajectory turns out to be plane, and rotation of the plane of polarization of light is absent.

An analogous conclusion concerning rotation of the plane of polarization can be drawn in the case of propagation of light in a weakly inhomogeneous moving dielectric. For this it is only necessary to replace the vector $\mathbf{g}$ by the vector $\mathbf{u}$:

$$\mathbf{u} = (1 - \varepsilon\mu)\mathbf{v}/c.$$

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Received
20 II 1967

CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.