ON AN ALGORITHM FOR USING INFORMATION REDUNDANCY IN MULTIDIMENSIONAL REPRODUCING SYSTEMS

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Abstract

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ON AN ALGORITHM FOR USING INFORMATION REDUNDANCY IN MULTIDIMENSIONAL REPRODUCING SYSTEMS

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1°. In paper (1), an iterative principle was formulated for constructing multichannel automatic-control systems with several meters of one and the same physical quantity, and the possibility was shown of compensating errors due to the useful signal and disturbances in such systems. We shall consider here the case of a multidimensional reproducing system; in contrast to (1), we assume that for each of the coordinates $x_i(t)$ there is not several, but only one reproduction (measurement) channel W_i, while the $x_i(t)$ themselves are homogeneous, sufficiently close, interrelated coordinates, in particular equal ones,

$$x_1(t) = x_2(t) = \dots = x_n(t) = x_0(t), \dots \tag{1}$$

of various objects. In this case the measured value $y_i(t)$ of one of the coordinates $x_i(t)$ contains considerable information about the remaining $x_1(t), \dots, x_{i-1}(t), x_{i+1}(t), \dots, x_n(t)$, which can be used to increase the accuracy of reproduction.

2°. Let us construct a multidimensional reproducing system in which, in determining the i-th coordinate, the results of measurements of all preceding coordinates would be used. The posed problem is satisfied by the recurrence relations corresponding to the iterative process of reproduction:

$$y_i(t) = y_{i-1}(t) + y_i^*(t), \tag{2}$$

$$y_i^*(t) = W_i(D)x_i^*(t), \tag{3}$$

$$x_i^*(t) = x_i(t) - y_{i-1}(t). \tag{4}$$

By successively using relations (2)—(4), one can find the matrix-operator $\mathcal{W}(D)$, transforming the input vector $x(t)$ into the output vector $y(t)$,

$$y(t) = \mathcal{W}(D)x(t), \tag{5}$$

$$\mathcal{W} = \left\| \begin{array}{ccccc} W_1 & 0 & 0 & \dots & 0 \\ W_1E_2 & W_2 & 0 & \dots & 0 \\ W_1E_2E_3 & W_2E_3 & W_3 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ W_1\prod_2^n E_\mu & W_2\prod_3^n E_\mu & W_3\prod_4^n E_\mu & \dots & W_n \end{array} \right\|, \tag{6}$$

where

$$E_\mu(D) = 1 - W_\mu(D). \tag{7}$$

From (2)—(4), or from (5)—(6), it is not difficult to see that the error $\varepsilon_i(t) = x_i(t) - y_i(t)$, for equal (1) or close input actions, will decrease as the number i of the channel W_i increases, i.e., the system is nonuniform in accuracy.

3°. It is natural to pose the problem of synthesizing such an operating algorithm under which, in the process of reproduction of any one of the coordinates, information about all the others would be used, ensuring potentially equal accuracy for each of the inputs $x_i(t)$.

Let w_{ij}^{p} denote the elements of the operator matrix $\mathcal{W}^{\mathrm{p}}$ of the desired equiprecise system, and w_{ij}^{n} those of the non-equiprecise iterative system (6). It is obvious that for $j \leq i$ one should take $w_{ij}^{\mathrm{p}} = w_{ij}^{\mathrm{n}}$. To determine the unknown functions $w_{ij}^{\mathrm{p}} = \gamma_{ij}$ for $j > i$, we shall require fulfillment of the condition that the transition in the matrix $\mathcal{W}^{\mathrm{n}}$ from the i-th to the $(i+1)$-st row be equivalent to the transition in the matrix $\mathcal{W}^{\mathrm{p}}$, for the same row, from the j-th column to the $(j+1)$-st (which in both cases corresponds to equal efficiency of the additional use of measurement results for one of the coordinates). Complete equivalence is meaningful when the signals (1) are equal. In this case the sum of the elements of the i-th row of matrix (6) determines the operator $\mathcal{W}_i^{\mathrm{n}}(D)$, transforming $x_i(t) = x_0(t)$ into $y_i(t)$:

$$\mathcal{W}_i^{\mathrm{n}}(D) = \sum_{j=1}^{i} w_{ij}^{\mathrm{n}}(D) = \sum_{j=1}^{i-1} W_j(D) \prod_{\mu=j+1}^{i} E_\mu(D) + W_i(D). \tag{8}$$

In passing from the i-th to the $(i+1)$-st row, as is seen from (8), in the non-equiprecise system the following recurrence relation holds:

$$\mathcal{W}_{i+1}^{\mathrm{n}}(D) = \mathcal{W}_i^{\mathrm{n}}(D) + W_{i+1}(D) - \mathcal{W}_i^{\mathrm{n}}(D)W_{i+1}(D). \tag{9}$$

Let $\Phi_{ik}(D)$ denote the sum of the first k elements of the i-th row of the equiprecise system,

$$\Phi_{ik}(D) = \sum_{j=1}^{k} w_{ij}^{\mathrm{p}}(D) = \sum_{j=1}^{i-1} w_{ij}^{\mathrm{n}}(D) + W_i(D) + \sum_{j=i+1}^{k} \gamma_{ij}(D), \tag{10}$$

and, as noted above, require that for $\Phi_{ik}(D)$ a recurrence relation analogous to (9) be satisfied:

$$\Phi_{i,k+1}(D) = \Phi_{ik}(D) + W_{k+1}(D) - \Phi_{ik}(D)W_{k+1}(D). \tag{11}$$

Then from (10), (11) for the unknown $\gamma_{ij}(D)$ we find

$$\gamma_{ij}(D) = W_j(D) \prod_{\mu=1}^{j-1} E_\mu(D), \qquad j > i. \tag{12}$$

Thus, the operating algorithm of the equiprecise multidimensional system is described by the matrix equation

$$y(t) = \mathcal{W}^{\mathrm{p}}(D)x(t), \tag{13}$$

$$\mathcal{W}^{\mathrm{p}} = \left\| \begin{matrix} W_1 & W_2E_1 & W_3E_1E_2 & \dots & \dots & W_n \prod_1^{n-1} E_\mu \\ W_1E_2 & W_2 & W_3E_1E_2 & \dots & \dots & W_n \prod_1^{n-1} E_\mu \\ W_1E_2E_3 & W_2E_3 & W_3 & \dots & \dots & W_n \prod_1^{n-1} E_\mu \\ \dots & \dots & \dots & \dots & \dots & \dots \\ W_1 \prod_2^{n} E_\mu & W_2 \prod_3^{n} E_\mu & W_3 \prod_4^{n} E_\mu & \dots & \dots & W_n \end{matrix} \right\|. \tag{14}$$

The analytic construction of structural schemes of multidimensional iterative systems satisfying algorithms (5)–(6) or (13)–(14) can be carried out by introducing additional require-

conditions, the most important of which is the invariance of the iterative algorithm with respect to changes in the parameters of the individual channels.

4°. We define the reproduction error $\varepsilon_i(t) = x_i(t) - y_i(t)$ from the matrix equation (13)–(14):

$$\varepsilon_i(t) = E_i(D)x_i(t) - \sum_{j=1}^{i-1} W_j(D) \prod_{\mu=j+1}^{i} E_\mu(D)x_j(t)$$
$$- \sum_{j=i+1}^{n} W_j(D) \prod_{\mu=1}^{i-1} E_\mu(D)x_j(t). \quad (15)$$

Denoting by ξ_{ik} the coordinate difference

$$\xi_{ik}(t) = x_i(t) - x_k(t), \qquad \xi_{ik}(t) = -\xi_{ki}(t), \quad (16)$$

from (15)–(16), for a three-channel system we find:

$$\begin{aligned} \varepsilon_1(t) &= E_1(D)E_2(D)E_3(D)x_1(t) + W_2(D)E_1(D)\xi_{12}(t) + \\ &\quad + W_3(D)E_1(D)E_2(D)\xi_{13}(t), \\ \varepsilon_2(t) &= E_1(D)E_2(D)E_3(D)x_2(t) + W_1(D)E_2(D)\xi_{21}(t) + \\ &\quad + W_3(D)E_1(D)E_2(D)\xi_{23}(t), \\ \varepsilon_3(t) &= E_1(D)E_2(D)E_3(D)x_3(t) + W_1(D)E_2(D)E_3(D)\xi_{31}(t) + \\ &\quad + W_2(D)E_3(D)\xi_{32}(t). \end{aligned} \quad (17)$$

In the most favorable case (1), when $\xi_{ik}(t) = 0$,

$$\varepsilon_i(t) = E_1(D)E_2(D)E_3(D)x_i(t), \quad (18)$$

i.e., for equal input signals the reproduction error does not depend on the index i, and the error operator of the multichannel equivalent system is equal to the product of the error operators of the individual channels. When $\xi_{ik}(t) \neq 0$, additional error components arise; the influence of the nonidentity $\xi_{ik}(t)$ of the coordinates on the accuracy of the iterative system is determined by formulas (17).

Assuming each of the channels to be first-order astatical ($W_i(0) = 1$), with velocity gain coefficient k_{vi}, the error operators $E_i(D)$ can be represented in the form

$$E_i(D) = \frac{D}{k_{vi}} E_i^*(D), \qquad \text{where } E_i^*(0) = 1. \quad (19)$$

Then, according to (17) ÷ (19):

$$\varepsilon_1(t) = \varepsilon_1^0(t) + W_2(D)E_1^*(D)\frac{\dot{\xi}_{12}(t)}{k_{v1}} + W_3(D)E_1^*(D)E_2^*(D)\frac{\ddot{\xi}_{13}(t)}{k_{v1}k_{v2}},$$
$$\varepsilon_2(t) = \varepsilon_2^0(t) + W_1(D)E_2^*(D)\frac{\dot{\xi}_{21}(t)}{k_{v2}} + W_3(D)E_1^*(D)E_2^*(D)\frac{\ddot{\xi}_{23}(t)}{k_{v1}k_{v2}}, \qquad (20)$$
$$\varepsilon_3(t) = \varepsilon_3^0(t) + W_2(D)E_3^*(D)\frac{\dot{\xi}_{32}(t)}{k_{v3}} + W_1(D)E_2^*(D)E_3^*(D)\frac{\ddot{\xi}_{31}(t)}{k_{v2}k_{v3}},$$

where

$$\varepsilon_i^0(t) = E_1^*(D)E_2^*(D)E_3^*(D)\frac{\dddot{x}_i(t)}{k_{v1}k_{v2}k_{v3}}. \qquad (21)$$

It is evident from this that the first component of the dynamic error $\varepsilon_i^0(t)$ is invariant with respect to the position, velocity, and acceleration of the coordinates $x_i(t)$, $\dot{x}_i(t)$, $\ddot{x}_i(t)$, while the second and third depend on the velocity and acceleration of the deviations $\xi_{ik}(t)$, $\dot{\xi}_{ik}(t)$. The errors are smaller, the higher the coefficients

channel gains. In a first approximation, for slowly varying signals, when calculating the steady-state errors in (20)-(21), one may take $E_i^*(D) = W_i^*(D) = 1$.

For comparison we give the expression for the error under independent measurements

$$\varepsilon_i(t) = E_i(D)x_i(t), \qquad (22)$$

or, taking (19) into account,

$$\varepsilon_i(t) = E_i^*(D)\dot{x}_i(t)/k_{vi}. \qquad (23)$$

Thus, the iterative algorithm for using information redundancy makes it possible to substantially increase the accuracy of reproduction by a multidimensional system of homogeneous, sufficiently close coordinates.

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References

1. P. F. Osmolovskii, DAN, 181, No. 1 (1968).

Note: Figure translations are in progress. See original paper for figures.