EMPIRICAL MASS REGULARITIES OF RESONANCE STATES

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Abstract

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In studying various processes of low-energy scattering and comparing their characteristics with one another, the question of choosing an energy variable common to the various processes is an essential one.

In dispersion analysis (see, for example, [1]) the following scale proves convenient:

$$x = s - m_i^2 - M_i^2, \tag{1}$$

where m_i is the mass of the incident particle and M_i is the mass of the target in the i-th reaction. For the case of forward scattering at $t = 0$, the replacement $x \to -x$ corresponds to the crossing transformation $s \leftrightarrow u$, as a result of which the scale (1) is especially convenient for comparing total cross sections.

We shall now show that the variable (1) also makes it possible to obtain a simple classification of two-particle resonances.

Table 1

Meson-meson resonances

	$\pi + \pi$	$\pi + K$	$K + \bar{K}$	$\bar{x}$
$P(1^-)$	$\rho(765)1^+(1^-)0.5$[illegible]$\pm$ 0.09	$K^*(890)1/2\,(1^-)0.54\pm$ 0.04	$\varphi(1020)0^-(1^-)0.5$[illegible]$\pm$ 0.004	0.55
$D(2^+)$	$f(1260)0^+(2^+)1.53\pm$ 0.15	$K_V(1420)1/2\,(2^+)1.75\pm$ 0.13	$f'(1514)0^+(2^+)1.8$[illegible]$\pm$ 0.11	1.70
$F(3^-)$	$\rho_V(1650)1^+(V)2.$[illegible]$2\pm$ 0.28	$K_A(1800)1/2\,(A)2.91\pm$ 0.13		2.80

Table 1 gives 8 resonance states decaying into two mesons $(M + M)$. At the bottom of each cell is given the value of the parameter x, calculated from the formula

$$x_i = M_{ik}^2 - m_i^2 - M_k^2, \qquad (2)$$

where M_{ik} is the mass of the resonance decaying into the particles m_i and M_k. The numerical values were taken from the latest review by Rosenfeld et al. [2]. It is evident from the table that the parameter x varies little along each horizontal row and thus turns out to be a good spectroscopic characteristic.

Table 2 contains the "x-spectroscopy" of 18 meson-baryon resonances and 3 bound states. In assigning the resonances and calculating x we always used the principal mode of decay. From the analysis of Table 2 it follows that x is a good spectroscopic characteristic also for $M + B$ resonances. Just as in Table 1, its values deviate from the mean values, given in the last column, by no more than $\pm 10\%$, which, with a few exceptions, lies within the limits of the experimental errors indicated in [2]. It should be noted that

s-wave resonances $N(1550)$, $N'(1710)$, $\Delta(1640)$, $\Lambda(1405)$, $\Lambda'(1670)$, as well as the Roper resonance $N'(1470)$, do not fit into the simple level scheme of Table 2. At the same time, the 3-pion resonances $\omega(783)$ and $A_2(1300)$ fit into Table 1. Here the coordinate x is determined by the formula $x = M_{\text{res}}^2 - 3\mu_\pi^2$.

The observed constancy of the parameter x over the multiplets J^P seems interesting to us in the following respects:

1) It makes it possible to express the horizontal distances between Regge trajectories a_{ik} in terms of differences of the squared masses of the products of the dominant decay mode m_i, M_k:

$$\Delta(a_{ik} - a_{ln}) = m_l^2 + M_n^2 - m_i^2 - M_k^2. \qquad (3)$$

Table 2

	$\pi + N$	$\overline{K} + N$	$\pi + \Lambda$	$\overline{K} + \Lambda$	$\pi + \Sigma$	$\pi + \Xi$	$\overline{x}$
$P\,(1/2^+)$		$\Lambda 0\,(1/2^+) 0,11$	$\Sigma 1\,(1/2^+) 0,16$	$\Xi 1/2\,(1/2^+) 0,25$			$0,17$
$P\,(3/2^+)$	$\Delta(1236) 3/2\,(3/2^+) 0,63 \pm 0,15$		$\Sigma(1385) 1\,(3/2^+) 0,66 \pm 0,05$			$\Xi(1530) 1/2\,(3/2^+) 0,58 \pm 0,01$	$0,62$
$D\,(3/2^-)$	$N(1518) 1/2\,(3/2^-)$ [illegible] $\pm 0,18$	$\Lambda(1520) 0\,(3/2^-)$ [illegible] $\pm 0,02$	$\Sigma(1660) 1\,(3/2^-) 1,50 \pm 0,03$		$\Lambda'(1690) 0\,(3/2^-) 1,42 \pm 0,08$		$1,38$
$D\,(5/2^-)$	$N(1680) 1/2\,(5/2^-) 1,92 \pm 0,29$	$\Sigma(1770) 1\,(5/2^-) 2,01 \pm 0,16$		$\Xi(1816) 1/2$ [illegible] $\pm 0,03$	$\Lambda(1830) 0\,(5/2^-)$ [illegible] $\pm 0,14$	$\Sigma(1930)$ [illegible] $\pm 0,27$	[illegible]
$F\,(5/2^+)$	$N(1688) 1/2\,(5/2^+)$ [illegible] $\pm 0,22$	$\Lambda(1815) 0\,(5/2^+)$ [illegible] $\pm 0,14$	$\Sigma(1910) 1\,(5/2^+) 2,39 \pm 0,11$				$2,20$
$F\,(7/2^+)$	$\Delta(1920) 3/2\,(7/2^+) 2,90 \pm 0,43$		$\Sigma(2030) 1\,(7/2^+) 2,86 \pm 0,24$				$2,88$
$G\,(7/2^-)$	$N(2190) 1/2\,(7/2^-)$ [illegible] $\pm 0,55$	$\Lambda(2100) 0\,(7/2^-) 3,23 \pm 0,29$					$3,61$

Consequently, all M + M Regge trajectories are described by a single functional dependence (not necessarily linear) $\alpha_{ik}^{\mathrm{MM}}(s) = a_{\mathrm{MM}}(x)$. In the linear approximation, for example, the meson-meson trajectories have the form:

$$\alpha_{\mathrm{MM}}^{ik}(s) = a_{\mathrm{MM}}(x) = 0,89(s - m_i^2 - M_k^2) + 0,50. \quad (4)$$

The two M + B trajectories $\alpha_{J+1/2}^{ik}(s)$ and $\alpha_{J-1/2}^{ik}(s)$ differ from each other by their slopes.

2) It makes it possible to obtain mass formulas relating the masses of resonances within multiplets to the squared masses of stable particles. These formulas are quadratic in the masses both for M + M and for M + B resonances.

3) It makes it possible to formulate dynamical equations of dispersion type in terms of the variable x simultaneously for all reactions of the form M + M and M + B.

4) It suggests that the parameter x plays the role of an eigenvalue of an operator describing the relative motion of two particles. A remarkable circumstance is the fact that the potential of this interaction practically does not depend on the kind of interacting particles. Its eigenvalues, with an accuracy of $\pm 10\%$ (which approximately coinc–

gives, within the accuracy of the experimental determination of the masses of resonant states, constants within each of the multiplets, determined by the total angular momentum J and the space parity P. Thus one may speak of the "levels" of multiplets $x(J^P)$. In Fig. 1 the levels of the systems $M + M$ and $M + B$ are shown. The approximate equidistance of the levels corresponds to the approximate linearity of the Regge trajectories. The mean spacing

$$x(J + 1^{\pm}) - x(J^{\pm}) \sim 1 \text{ GeV}^2$$

indicates that these levels may be regarded as quasistationary levels whose wave functions can be characterized by an interaction radius

$$R_{\text{eff}} \sim 1 \text{ GeV}^{-1} \sim 2 \cdot 10^{-14} \text{ cm}.$$

The spin splittings of the levels of the $M + B$ system

$$x(J + {}^1/{}^{\pm}_2) - x(J - {}^1/{}^{\pm}_2) \simeq (0.4 \div 0.7) \text{ GeV}^2$$

increase only weakly with increasing J. The level $P({}^1/{}^{+}_2)$ is stable. A striking feature of both spectra is the absence of s-levels.

Fig. 1. Spectra of meson-meson and meson-baryon resonant multiplets in the variable $x = M_{\text{res}}^2 - m_i^2 m_k$ in units of $(\text{GeV})^2$. The level $P({}^1/{}^{+}_2)$ is stable.

In the following note we shall try to give an outline of a theory of resonant states leading to spectra of the type obtained.

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Note added in proof. 1. Taking into account $SU(3)$-symmetry, which leads to the separation of the unitary singlets $\Lambda(1520)$ and $\Lambda(2100)$ from the multiplets $({}^3/{}^-_2)$ and $({}^7/{}^-_2)$, substantially reduces the scatter of the values of the parameter x within these multiplets. As a result it turns out that, for reliably established members of the octets $({}^3/{}^-_2)$, $({}^5/{}^-_2)$ and decuplets $({}^3/{}^+_2)$, $({}^7/{}^+_2)$, this scatter lies within ± 0.05 GeV^2, i.e., amounts to about $\pm 5\%$ on the x scale, quadratic in the masses.

2. Experimental data on the s-resonances $N(1550)$, $\Lambda(1670)$, $\Delta(1640)$, $N(1710)$, apparently indicate the presence of at least one $S({}^1/{}^-_2)$-level, lying in the region $x \sim 1.5$ GeV, i.e., in the vicinity of the D-levels.

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REFERENCES

1. V. V. Serebryakov, D. V. Shirkov, *Nuovo Cimento*, **51A**, 878 (1967).
2. A. H. Rosenfeld et al., *Rev. Mod. Phys.*, **40**, 77 (1968).

Note: Figure translations are in progress. See original paper for figures.