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Abstract

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THE INFLUENCE OF FINITE CONDUCTIVITY ON STEADY SELF-COMPRESSING PLASMA FLOWS

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In papers ($^{1-3}$) the authors* described and analyzed steady axially symmetric plasma flows accompanied by compression under the action of their own azimuthal magnetic field. These flows were called self-compressing (^{3}). In paper (^{3}), for an ideal one-component plasma, extremal values were found for the density ρ_M, temperature T_M, and magnetic field H_M, attainable in such flows in the absence of external magnetic fields.

In particular, the maximum value of the degree of compression proved to be equal to

$$\frac{\rho_M}{\rho_0}\left(\frac{c_{A0}^2}{c_{T0}^2}(\gamma-1)+1\right)^{1/(\gamma-1)}, \qquad \gamma>1,$$
$$\frac{\rho_M}{\rho_0}=\exp\left(\frac{c_{A0}^2}{c_{T0}^2}\right), \qquad \gamma=1. \tag{1}$$

Here ρ_0, c_{T0}, and c_{A0} are the values of the density, the sound speed, and the Alfvén speed, taken at the start of the flow, where the plasma velocity $v=0$; γ is the adiabatic exponent.

However, the calculations given in the cited papers were made for an ideally conducting plasma and under fairly severe restrictions of a geometrical character. The present work, carried out with the aid of a digital computer, is devoted primarily to elucidating the general picture of an axially symmetric self-compressing magnetohydrodynamic flow and the influence on it of the ohmic resistance of the plasma. At the same time, the time of establishment is found and the stability of such flows with respect to axially symmetric perturbations is shown. In the calculations the plasma is assumed to be inviscid and to have constant conductivity. The Hall effect is neglected. Instead of the energy equation, the polytropic equation is used.

The system of dimensionless equations describing the plasma flow with the indicated properties has the form:

Fig. 1

Figure 1: Fig. 1

$$\partial\rho/\partial t + \operatorname{div}\rho\mathbf{v} = 0, \qquad \rho(\partial\mathbf{v}/\partial t + (\mathbf{v}\nabla)\mathbf{v}) = -\nabla p + \operatorname{rot}\mathbf{H}\times\mathbf{H},$$
$$\partial\mathbf{H}/\partial t = \operatorname{rot}[\mathbf{v},\mathbf{H}] + \frac{1}{R_m}\Delta\mathbf{H}, \qquad p = \beta\rho^\gamma, \tag{2}$$
$$\mathbf{H} = (0, H, 0), \qquad \mathbf{v} = (v_r, 0, v_z).$$

The parameters determining the flows enter equations (2) in the form of two dimensionless combinations

$$\beta = 4\pi p_0/H_0^2, \qquad R_m = 4\pi\sigma L a_0/c^2. \tag{3}$$

* Paper ([2]) was carried out jointly with L. S. Solov' ev.

Here ρ_0, p_0, H_0 are the characteristic density, pressure, and magnetic field at the entrance to the channel, L is the channel length, c is the speed of light, and σ is the conductivity. As the velocity scale the quantity

$$a_0 = H_0/\sqrt{4\pi\rho_0}.$$

is taken.

The system of equations (2) is solved by the relaxation method. Solutions of system (2) found for a number of cases have been presented earlier in works ([4],[5]).

Fig. 1

In (3) it was shown that the extremal value of the density is attained in the case when the distance from the flow tube to the system axis tends to zero. Therefore, for the calculations it is natural to take the system shown in Fig. 1*. For simplicity we approximate the electrodes by parabolas

$$r_1 = 0.125 - 2z^2, \qquad r_2 = 0.2z^2 - 0.2z + 0.175. \tag{4}$$

The boundary conditions at the entrance ($z = 0$) and exit ($z = 1$) of the system are specified in the form ([1],[6])

$$\rho\big|_{z=0} = \rho_0 = \text{const}, \qquad H\big|_{z=0} = H_0\frac{r_0}{r}, \qquad \frac{\partial}{\partial z}Hf\big|_{z=1} = 0. \tag{5}$$

As boundary conditions on the electrode surfaces we take equality to zero of the normal component of the velocity and of the tangential component of the electric field

$$v_n\Big|_{r_1,r_2} = 0, \qquad E_t\Big|_{r_1,r_2} = 0. \tag{6}$$

The initial conditions are chosen in such a way as to ensure acceleration of the plasma along the channel ([1],[6]).

Analysis of the computational data leads to the following conclusions:

1. Stable axially symmetric flows are established over a time of the order of the flight time ($\sim L/v_0$).
2. The flows are accompanied by compression, and the maximum density values ρ^* agree well with the values calculated, for known ρ_0, r_0, H_0 and velocities at the entrance v_0 and in the region of maximum compression v^*, from the Bernoulli equation

$$v^2/2 + W(\rho) + H^2/4\pi\rho = \text{const.} \tag{7}$$

Here $W(\rho)$ is the enthalpy of the plasma. The observed values of ρ^* are, in order of magnitude, comparable with ρ_M (see (1)).

The validity of (7) in estimating ρ^* was observed for $0.1 < R_m < 1000$. Physically this is easy to explain if one takes into account that, for finite plasma conductivity and $\mathbf{j} \perp \mathbf{v}$, the Bernoulli integral in the form ([7]) remains valid

$$\left[\rho v\left(v^2/2 + W\right) + cEH/4\pi\right] f = \text{const.} \tag{8}$$

Here f is the width of the flow tube. On the surface of the central electrode the condition $\mathbf{j} \perp \mathbf{v}$ is fulfilled by virtue of (6), while near the entrance, because of the low current density,

$$E \approx \frac{v}{c} H. \tag{9}$$

* This system was called a magnetoplasma compressor.

Large compression ratios, obtained in the isothermal regime at $\beta < 1$, led to overflow of the digital computer. Therefore, flows with $\beta \sim 0.1$ could be investigated in detail only for adiabatic regimes ($\gamma = 5/3$).

In Fig. 2, solid lines show lines of constant density for $\gamma = 5/3$ and two values of the conductivity, $R_m = 1$ and 1000, for

Fig. 2

Fig. 2

Figure 2: Fig. 2

Fig. 3

Figure 3: Fig. 3

$\beta = 0.1$ and 0.3 (a—$\beta = 0.1$, $R_m = 1000$; —$\beta = 0.1$, $R_m = 1$; —$\beta = 0.3$, $R_m = 1$). It is seen that near the end of the central electrode a "focus" is formed, into which the plasma flow converges and from which it then diverges. At some distance from the end of the central electrode a second, but substantially weaker, focus is formed.

3. Ohmic resistance has a strong effect only on the distribution of currents in the system. If R_m is small*, then the current rapidly leaves the compressing plasma, and in the focus zone the plasma proves to be completely currentless. If, however, R_m is large, then the plasma in the compression zone also proves to be magnetized.

Fig. 3

In Fig. 2 the electric-current lines are shown by dashed lines. Noteworthy is the formation, at large R_m, of current vortices near the outer electrode. A current vortex is also formed near the focus.

* According to preliminary estimates, R_m is small if $R_m < L^2/\delta^2$, where δ is a certain characteristic width of the compressed state.

4. Figure 3 shows the behavior of the velocity component $u \equiv v_z$ along the coordinate lines shown in Fig. 1. It is seen that, near the first focus, where all the parameters change very sharply, deceleration of the flow also occurs. One may attempt to interpret the neighborhood of the first focus as a shock-wave zone. It should be noted that, in self-compressing flows, the formation of shock waves is in principle not necessary ([3]).

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Note: Figure translations are in progress. See original paper for figures.