COMPOSITE FIELDS OF INFINITE MULTIPLETS

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COMPOSITE FIELDS OF INFINITE MULTIPLETS

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As a relativistic generalization of the symmetry group $SU(6)$, Budini and Fronsdal and Michel proposed the symmetry group $G = P \cdot S$, which is a semidirect product of the Poincaré group P and a noncompact internal-symmetry group S, containing the subgroup $SL(2, C)$. In such a theory with infinite multiplets the symmetry properties and the unitarity condition for the S-matrix are compatible.

In the works of Feldman and Matthews ([1]), Nguyen Van Hieu and Fronsdal ([3]), various attempts were made to introduce quantized fields describing these infinite multiplets. To describe each (infinite) multiplet, the authors of ([1]) proposed using a field operator that forms a unitary irreducible representation of a certain auxiliary group isomorphic to S. This field operator carries indices running over an infinite set of values. For such a field operator one can write only one wave equation—the Klein–Gordon equation—regardless of whether the corresponding particles have integer or half-integer spins. Then all particles must obey Bose-Einstein statistics independently of their spins. Another method was proposed in ([2,3]). According to this method, in order to describe each infinite multiplet it is necessary to introduce an infinite number of spinor fields transforming according to nonunitary finite-dimensional representations of the auxiliary group and satisfying the Bargmann–Wigner equation (component or projective physical fields).

Naturally the question arises whether, within the framework of the second of the methods mentioned, one can introduce a large field. Will such a field then be local simultaneously with the component fields? Moreover, will the field energy obtained from the Lagrangian be positive definite? Here we follow the method proposed by one of us in ([2]). For simplicity we consider the symmetry group $SL(2, C)$. We note that the large field was also considered in ([3]).

As is known, for the classification of elementary particles it is necessary to construct a basis of representations corresponding to the reduction $SL(2, C) \supset SU(2)$, while for physical moving particles it is necessary to construct a basis corresponding to the reduction $SL(2, C) \supset SU(2)_p$. The explicit form of the bases for both reductions was given in [4,5]: the basis for the first reduction is formed from the generalized spinors of $SU(2)$

$$\Phi^{b_1 \ldots b_{j-\nu}}_{a_1 \ldots a_{j+\nu}}(z)$$

(the sign $\sim$ means that the corresponding indices refer to the system in which the particles are at rest), and the basis for the second reduction is formed from spinors of the form

$$\Phi^{b_1 \ldots b_{j-\nu}}_{a_1 \ldots a_{j+\nu}}(p;z),$$

which transform according to the corresponding spinor representations of the homogeneous Lorentz group. With the aid of the momenta

$$\left(-\frac{i}{m}\hat{p}\right)^{c}_{\dot{b}}$$

one can transform the dotted indices of the spinor

$$\Phi^{b_1 \ldots \dot{b}_{j-\nu}}_{a_1 \ldots a_{j+\nu}}(p;z)$$

into undotted indices. In addition, all upper indices can

lower by multiplication by the antisymmetric spinor ε_{ab}. As a result we obtain symmetric spinors, which for brevity we denote by $\Phi_{\widetilde{a_{2j}}}(z) \equiv \Phi_{\widetilde{a_1 \ldots a_{2j}}}(z)$, $\Phi_{a_{2j}}(p;z) \equiv \Phi_{a_1 \ldots a_{2j}}(p;z)$. Along with each spinor $\Phi_{a_{2j}}(p;z)$ one may also introduce $2^{2j}-1$ spinors with all possible numbers of lower dotted indices by multiplying $\Phi_{a_{2j}}(p;z)$ by the required number of momenta $\left(-\frac{i}{m}\hat{p}\right)^{a}_{\dot{c}}$. These spinors, taken together, form the spinor $\Phi_{\alpha_{2j}}(p;z)$, where α_i now assumes four values $1, 2, \dot{1}, \dot{2}$. Instead of the basis $\Phi_{\widetilde{a_{2j}}}(z)$ one may use the basis $\xi_{jj_3}(z)$, which is a linear combination of $\Phi_{\widetilde{a_{2j}}}(z)$ for a fixed value of j. Between the bases $\xi_{kk_3}(z)$ and $\Phi_{\alpha_{2j}}(p;z)$ there is a simple relation

$$\Phi_{\alpha_{2j}}(p;z) = \sum_{kk_3} D_{\alpha_{2j}}(p;kk_3)\xi_{kk_3}(z).$$

The matrix $D_{\alpha_{2j}}(p;kk_3)$ is, up to certain transformations, the matrix of a finite Lorentz transformation carrying the particle from the state of rest into the state with 4-momentum p [5].

In order that the theory possess "crossing symmetry" (by crossing symmetry one should understand the possibility of applying the Lüders substitution rule), in addition to the basis $\Phi_{\alpha_{2j}}(p;z)$, we introduce also the basis $\Phi_{\alpha_{2j}}(-p;z)$, obtained from $\Phi_{\alpha_{2j}}(p;z)$ by the substitution $p \to -p$. In the present work we use only self-conjugate representations $\tau \sim (\nu, 0)$.

From the orthonormality condition of the basis vectors $\xi_{jj_3}(z)$ it is not difficult to obtain:

$$D_{\alpha_{2j}}(\pm p; kk_3) = \int \Phi_{\alpha_{2j}}(\pm p; z)\xi^{+}_{kk_3}(z)\, d\mu(z), \tag{1}$$

where $d\mu(z)$ is the invariant measure on the group [5].

The matrix $D_{\alpha_{2j}}(\pm p; kk_3)$ satisfies the following relations, whose proofs are omitted here:

$$D_{\alpha_1\ldots\alpha_i\ldots\alpha_{2j}}(\pm p; kk_3)\,\frac{1}{2}\left(1 \mp \frac{1}{m}\hat{p}\right)^{\alpha_i}_{\beta_i} = D_{\alpha_1\ldots\beta_i\ldots\alpha_{2j}}(\pm p; kk_3), \tag{2}$$

$$D_{\alpha_{2j}}(\pm p; kk_3)\overline{D}^{\beta_{2j}}(\pm p; kk_3) = (\pm)^{2\nu}\frac{1}{2^{2j}}\left(1 \mp \frac{i}{m}\hat{p}\right)^{\beta_1}_{\alpha_1}\cdots\left(1 \mp \frac{i}{m}\hat{p}\right)^{\beta_{2j}}_{\alpha_{2j}}\delta_{ij}, \tag{3}$$

$$D_{\alpha_{2j}}(\pm p; kk_3)\overline{D}^{\alpha_{2j}}(\pm p; ii_3) = (\pm)^{2\nu}\delta_{ik}\delta_{i_3k_3}. \tag{4}$$

Let now ψ be some element of the Hilbert space realizing the self-conjugate representation $\tau \sim (\nu, 0)$. We regard ψ as a large field containing an infinite multiplet. One may expand ψ in the bases $\Phi_{\alpha_{2j}}(p;z)$ and $\Phi_{\alpha_{2j}}(-p;z)$ in the following way:

$$\psi(p;z) = \sum_{j_3jk_3} u^{\alpha_{2j}}(p;j_3)a(p;jj_3)D_{\alpha_{2j}}(p;kk_3)\xi_{kk_3}(z) +$$

$$+\, v^{\alpha_{2j}}(-p;j_3)b^{+}(p;jj_3)D_{\alpha_{2j}}(-p;kk_3)\xi_{kk_3}(z), \tag{5}$$

where a, b^{+} are, respectively, the operators of annihilation of a particle and creation of an antiparticle.

Let us now write the general field (5) in the x-representation:

$$\psi(x;z) = \int \sum_{j_3kk_3} u^{\alpha_2 j}(p;j_3)\, a(p;jj_3)\, D_{\alpha_2 j}(p;kk_3)\, \xi_{kk_3}(z)e^{-ipx} +$$

$$+\, v^{\alpha_2 j}(-p;\bar{j}_3)\, b^{+}(p;jj_3)\, D_{\alpha_2 j}(-p;kk_3)\, \xi_{kk_3}(z)e^{ipx}\, d\mu(p), \tag{6}$$

where $d\mu(p)$ is the usual invariant measure on the mass surface.

Assuming that the operators a, b satisfy the usual commutation relations and using (2), (4) and the generalized summation formulas over the spin index for $u^{\alpha_2 j}$, $v^{\alpha_2 j}$, we obtain

$$[\psi(x;z), \psi^+(y;w)]_\pm = \delta(z-w) \int \{[e^{-ip(x-y)} \pm e^{ip(x-y)}]\, d\mu(p)\}. \tag{7}$$

Thus, the causality condition requires that the large field be quantized according to Bose statistics, independently of its spin content. This result was obtained by Feldman and Matthews ([1]).

Integrating the energy density of the large field, we obtain

$$P^0 = \iint \left[\sum_k \frac{\partial\psi^+}{\partial x^k}\frac{\partial\psi}{\partial x^k} - m^2\psi^+\psi\right] d\mu(z)\, dx =$$

$$= \int dp\, p^0 \sum_{ii_3} [a^+(p;ii_3)a(p;ii_3) + b(p;ii_3)b^+(p;ii_3)]\,. \tag{8}$$

It is clear from (8) that after quantization of the large fermion field the energy is positive definite. In order to achieve positive definiteness of the energy, we must introduce, instead of the field $\psi^+(y;w)$, the field $\widetilde{\psi}(y;w) = [\varepsilon(p^0)]^{2\nu}\psi^+(y;w)$, which differs from the field $\psi^+(y;w)$ by the factor $(-)^{2\nu}$ in the second term. Instead of (7) we now have

$$[\psi(x;z), \widetilde{\psi}(y;w)]_\pm = \delta(z-w) i\Delta(x-y),$$

and here one must take the minus sign for bosonic fields and the plus sign for fermionic fields. In the theory constructed from ψ and $\widetilde{\psi}$, the energy is positive definite and the correct relation between spin and statistics holds; however, the locality of the large field is violated because of the operator $[\varepsilon(p^0)]^{2\nu}$. A similar result was obtained by Fronsdal ([3]).

Let us now consider the interaction of an infinite multiplet with some singlet field. The operator matrix element has the form

$$F(s,t,u) \int \widetilde{\psi}(p_2;z)\psi(p_1;z)\, d\mu(z), \tag{9}$$

where $F(s,t,u)$ is some form factor. In expression (9) the field operators of the singlet field have been omitted. Substituting (6) into (9), we obtain for the scattering channel and the annihilation channel the matrix elements

$$\sum_{jk_3} \bar{u}_{\alpha_2 j}(p_2)\, \overline{D}^{\alpha_2 j}(p_2; kk_3)\, D_{\beta_2 j}(p_1; kk_3)\, u^{\beta_2 j}(p_1), \tag{10}$$

$$\sum_{jk_3} \bar{v}_{\alpha_2 j}(-p_2)\, \overline{D}^{-\alpha_2 j}(-p_2; kk_3)\, D_{\beta_2 j}(p_1; kk_3)\, u^{\beta_2 j}(p_1). \tag{11}$$

Expressions (10), (11) show in the most general form that the Lou replacement rule holds here. However, there are as yet no theoretical arguments for the analytic continuation of (10) into (11). For illustration, let us consider the annihilation process $1/2 + 1/2 \to 0 + 0$. We compute the explicit expression for the matrix element. Using (11), (1), ···

we obtain

$$\bar{v}_\alpha(-p_2) u^\beta(p_1) \int \bar{\Phi}^\alpha(-p_2; z) \Phi_\beta(p_1; z)\, d\mu(z) =$$
$$= \bar{v}_\alpha(-p_2) u^\alpha(p_1) \frac{1}{2(1-\alpha)\sqrt{\alpha^2-1}} \left[\sqrt{-\alpha + \sqrt{\alpha^2-1}} - \sqrt{-\alpha\sqrt{\alpha^2-1}} \right], \tag{12}$$
$$\left(\alpha = -\frac{p_1 p_2}{m^2} \right).$$

Expression (12) can indeed be obtained directly from the matrix element of the scattering process (see (28) in (4)) by Low’ s replacement rule $t \to s$.

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Note: Figure translations are in progress. See original paper for figures.