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Abstract

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CYBERNETICS AND CONTROL THEORY

I. A. GANIN

SUFFICIENT CONDITIONS FOR STABILITY IN PROBABILITY OF CONTROL SYSTEMS WITH VARIABLE STRUCTURE

(Presented by Academician B. N. Petrov on 19 IV 1968)

1. We consider automatic control systems with variable structure (see ([1], [2]), etc.), whose motion is described by the differential equation

$$dx/dt = F(x, u, t) + B(x, t)\xi(t). \tag{1}$$

Here x and F are vectors in E_n; $\xi(t)$ is a random process with values in E_k; $B(x, t)$ is an $n \times k$ matrix; $B(0, t) \equiv 0$. The components of the vector F have the form

$$F_i = x_{i+1} \quad (i = 1, \dots, n-2),$$

$$F_{n-1} = x_n - \sum_{i=1}^{n-1} c_i x_i, \tag{2}$$

$$F_n = (c_{n-1} - a_n)x_n - \sum_{i=1}^{n-1} x_i \left[c_i(c_{n-1} - a_n) - c_{i-1} + a_i\right] - u;$$

c_i are constant positive coefficients, a_i are piecewise-continuous bounded functions of time (object parameters).

The control function is formed as a linear combination

$$u = \sum_{i=1}^{s} \psi_i x_i + \rho x_n, \qquad 1 \le s \le n-1, \tag{3}$$

where

$$\psi_i = \begin{cases} \omega_i, & \text{for } x_i x_n > 0, \\ \lambda_i, & \text{for } x_i x_n < 0; \end{cases}$$

ρ is a nonnegative constant.

The solution of the equation

$$dx/dt = F(x, u, t)$$

will be understood in the sense of A. F. Filippov ([3]).

2. Introduce a positive definite quadratic form with constant coefficients

$$A(x) = \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} a_{ij} x_i x_j + a_{nn} x_n^2 \quad (a_{ij} = a_{ji}). \tag{4}$$

We also denote

$$\|B\| = \left(\sum_{i=1}^{n} \sum_{j=1}^{k} b_{ij}^2 \right)^{0.5}.$$

Lemma 1. Let $x^0(t, x_0, t_0)$ be a solution of equation (1). Suppose, furthermore, that the inequalities are valid

$$\begin{gathered} \omega_i > \sup_t [c_{i-1} - a_i - c_i(c_{n-1} - a_n)], \\ \lambda_i < \inf_t [c_{i-1} - a_i - c_i(c_{n-1} - a_n)], \\ \|B\| \leqslant \beta \sqrt{A(x)}. \end{gathered} \tag{5}$$

Then for the form (4) the following upper estimate holds:

$$A(x^0, t) \leqslant A(x^0, 0) \exp \left\{ Lt \left(\frac{\bar{\mu}}{L} + \frac{1}{t} \int_0^t |\xi(u)| \, du \right) \right\}, \tag{6}$$

Here $L > 0$ and $\bar{\mu}$ are certain constants.

Definition. The trivial solution of equation (1) is called **asymptotically stable in probability in the large** if, for any $\varepsilon > 0, \delta > 0$, there exists r such that

$$P\{|x(t, x_0, t_0)| > \varepsilon\} < \delta \quad \text{for } t \geqslant t_0,\ |x_0| < r$$

and, moreover, for any $x_0, \varepsilon > 0$,

$$P\{|x(t, x_0, t_0)| > \varepsilon\} \to 0 \quad \text{as } t \to \infty.$$

Theorem. Suppose that the conditions of the formulated Lemma 1 are satisfied. Then the trivial solution of the variable-structure system (1) is asymptotically stable in probability in the large if the process $|\xi(t)|$ satisfies the law of large numbers and

$$\bar{\mu}/L + \sup_t \mathbf{M}|\xi(t)| < 0. \tag{7}$$

Here $\mathbf{M}$ is the expectation operator.

The proof is based on the results of (4).

For a certain class of random processes, in particular for Gaussian processes, by averaging (6) one can obtain an estimate for the expectation of the form $A(x)$.

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Moscow Aviation Institute
named after S. Ordzhonikidze

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REFERENCES

1. S. V. Emel' yanov, V. I. Utkin, DAN, **152**, No. 2 (1964).

2. B. N. Petrov, S. V. Emel' yanov, M. B. Grineko, DAN, **169**, No. 1 (1966).

3. A. F. Filippov, Mat. sborn., **51**(93), No. 1 (1960).

4. R. Z. Khas' minskii, PMM, **30**, issue 5 (1966).

Note: Figure translations are in progress. See original paper for figures.