ELECTROMAGNETIC FORM FACTORS OF NUCLEONS IN A MODEL OF QUASI-INDEPENDENT QUARKS

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Abstract

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ELECTROMAGNETIC FORM FACTORS OF NUCLEONS IN A MODEL OF QUASI-INDEPENDENT QUARKS

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1°. One of the interesting problems considered from the point of view of higher symmetry groups was the explanation of the experimentally established relations between the normalized Sachs form factors of nucleons ([1])

$$G_{Ep}(q^2) = G_{Mp}(q^2) = G_{Mn}(q^2), \qquad G_{En}(q^2) = 0, \quad -q^2 \lesssim 100 f^{-2}. \quad (1)$$

However, symmetry considerations alone do not make it possible to establish all the relations (1). To establish a connection between the electric and magnetic form factors, it proves necessary to introduce certain additional dynamical considerations.

In papers ([2,3]), where a relativistic quark model of baryons was investigated, the electromagnetic form factors of nucleons were obtained under the assumption of a minimal electromagnetic interaction of quarks. This latter hypothesis corresponded to a model in which quarks have no intrinsic structure, while the structure of the composite particles (hadrons) is induced by the interaction of quarks. The electromagnetic current of the baryon octet in this model had the form

$$(J_\mu)_{88} = \left\{ \frac{p_\mu}{m} (\bar{\psi}\psi)_F \left(1 + \frac{q^2}{2m^2}\right) + \frac{1}{2m} (\bar{\psi} r_\mu \psi)_{3D+2F} \right\} F(q^2), \quad (2)$$

where p_μ is the momentum of the baryon center of mass, $F(q^2)$ is a relativistically invariant form factor specific to the model, normalized to unity at $q^2 = 0$, and

$$r_\mu = \frac{1}{m} \varepsilon_{\mu\nu\lambda\rho} q_\nu p_\lambda \gamma_\rho \gamma_5. \quad (3)$$

From the expression for the electromagnetic current (2), relations between the electromagnetic form factors follow, of the form

$$G_{Ep}(q^2) = (1 + q^2/2m^2)G_{Mp}(q^2), \qquad G_{Mp}(q^2) = G_{Mn}(q^2), \qquad G_{En}(q^2) = 0. \tag{4}$$

From consideration of (4) it is seen that the relations determined by group properties follow from this model, whereas the first, dynamical relation differs from the experimental data.

In papers ([4,5]) it was shown that, while remaining within the hypothesis of a minimal electromagnetic interaction of quarks, it is possible to ensure the correct form of the connection between the electric and magnetic form factors of the nucleon. For this it proves sufficient to assume that in collinear processes the magnetic field interacts with the W-spin operator ([6]), and not with the ordinary spin operator.

In covariant form this requirement corresponds to choosing the interaction of the i-th quark with the electromagnetic field in the form

$$\Gamma^{(i)} = \left[2e(AP) + \frac{i3e}{2m}\varepsilon_{\mu\nu\lambda\rho}F_{\mu\nu}p_\lambda\gamma_\rho\gamma_5\right]_{a_i}^{a_i'}, \tag{5}$$

moreover, indeed, in the rest system of the nucleon ($p = 0$) this expression has the form

$$2m\left[e\varphi + \frac{3e}{m}W\mathcal{H}\right]_{a_i}^{a_i'} \tag{6}$$

(a are unitary indices).

The electromagnetic current of the baryon octet could then be written as

$$(J_\mu)_{88} = \left\{\frac{p_\mu}{m}(\bar{\psi}\psi)_F + \frac{1}{2m}(\bar{\psi}r_\mu\psi)_{3D+2F}\right\}F(q^2), \tag{7}$$

whence the experimentally observed relations between the electromagnetic form factors (1) followed immediately.

It is necessary to note, however, that the introduction of the interaction in the form (5) was, in the approach described, only a hypothesis, which found some support in the known success of applying the group SU_W (6) to the description of collinear processes (7).

2°. The purpose of the present work is to obtain an expression of the form (5) within the framework of the model of quasi-independent quarks used by us (8–10).

Let us recall, in general outline, the basic assumptions of this model. We suppose that baryons are strongly bound states of three fundamental particles—quarks—which have a very large mass. This quark mass is largely compensated by a large binding energy in the baryon (the necessity of this follows from the stability of baryons with respect to quark decay), and effective quarks with a residual mass equal to one third of the baryon mass move in the resulting self-consistent field in a quasi-independent manner.

Assuming that the interaction between the effective (quasi-independent) quarks is completely switched off, we obtain the following equations for the baryon wave function:

$$(\gamma^{(i)} l^{(i)} - m)\Psi_{ABC} = 0, \qquad i = 1, 2, 3. \tag{8}$$

Here $l^{(i)}$ is the momentum of the i-th quark in the baryon. These equations express the requirement that the effective quarks be fermions.

It is easy to verify that the solution of equations (8) is

$$\Psi = \prod_{(i)} (1 + \gamma^{(i)} l^{(i)}/m)\, \Phi(p), \tag{9}$$

where $p = \sum l^{(i)},\ l^{(i)2} = m^2$.

Since we must require that the baryon in the model under consideration also be a fermion with spin 1/2, the center-of-mass wave function $\Phi(p)$ must be subjected to the additional condition (Bargmann–Wigner (12))

$$(\gamma^{(i)} p - m)\Phi(p) = 0. \tag{10}$$

Let us note that the wave function $\Phi(p)$, besides the center-of-mass momentum, also depends on the quark indices and on the invariants of the three-quark system.

We shall further assume (which is natural in the model of quasi-independent quarks) that the momentum transfer in a process in which a baryon participates occurs through one of its constituent quarks (say, the first), leaving the other quarks unaffected. Therefore let us introduce new variables convenient for the subsequent exposition:

$$L^{(j)} = \frac{l^{(j)} + l'^{(j)}}{2}, \quad j = 1, 2, 3; \qquad k = \sum_{(i)} (l^{(i)} - l'^{(i)}) = l^{(1)} - l'^{(1)}; \qquad L = \sum_{(j)} L^{(j)}, \tag{11}$$

whence

$$p' = L - k/2, \qquad p = L + k/2. \tag{12}$$

3°. We shall consider a normalized matrix element of the form

$$\int \bar{\widetilde{\Psi}} m \widetilde{\Psi}\, d\tau \Big/ \int \bar{\widetilde{\Psi}} \widetilde{\Psi}\, d\tau, \tag{13}$$

where

$$m = \sum_{(i)=1}^{3} A_\nu(x^{(i)})\gamma_\nu^{(i)}.$$

From the quantity (13) one can obtain the normalized electromagnetic form factors of baryons, if we correctly define the relativistically invariant matrix element (13).

The functions in (13) differ from the wave functions Ψ introduced earlier by additional factors expressing the Markov–Yukawa conditions [13–15], and we can thus write, in accordance with the prescription of work [9], the following explicit expression for the matrix element under consideration:

$$\begin{aligned}\int \bar{\widetilde{\Psi}} m \widetilde{\Psi}\, d\tau \equiv \sum_{(i)=1}^{3} \int \bar{\Psi}(l^{(1)}, l^{(2)}, l^{(3)}) A_\nu(k) \gamma_\nu^{(i)} \Psi(l'^{(1)}, l'^{(2)}, l'^{(3)}) \times \\ \times\, \delta(l'^{(j)} - l^{(j)} + k)\, \delta(l'^{(j_2)} - l^{(j_2)})\, \delta(l'^{(j_3)} - l^{(j_3)})\, \delta\Big(\sum l^{(i)} - p\Big)\, \delta\Big(\sum l'^{(i)} - p'\Big) \times \\ \times \prod_{j'=(j_2)}^{(j_3)} \delta\left[\left(l^{(j')} - \frac{p}{3}\right) p\right] \prod_{j''=(j_2)}^{(j_3)} \delta\left[\left(l'^{(j'')} - \frac{p'}{3}\right) p'\right] \prod_{i',\, i=1}^{3} dl^{(i)} dl'^{(i)} \equiv \sum_{(i)} \alpha^{(i)},\end{aligned} \tag{14}$$

where $j_2 \neq j_3 \neq j$; $j, j_2, j_3 = 1, 2, 3$, and the baryon wave functions Ψ are defined by formulas (9) and (10).

The normalization integral from formula (13) is defined as

$$\begin{aligned}\int \bar{\Psi}(l^{(1)}, l^{(2)}, l^{(3)}) \Psi(l^{(1)}, l^{(2)}, l^{(3)}) \delta\Big(\sum l^{(i)} - p\Big)\, \delta\left[\left(l^{(2)} - \frac{p}{3}\right) p\right] \times \\ \times \delta\left[\left(l^{(3)} - \frac{p}{3}\right) p\right] \prod_{(i)} dl^{(i)} = C.\end{aligned} \tag{15}$$

Let us choose (for convenience and without loss of generality) from the sum in (14) one term, for example

$$m^{(1)} = A_\nu(k)\gamma_\nu^{(1)}. \tag{16}$$

Substituting (9) and (16) into the formula for the matrix element (14), we obtain

$$\begin{aligned}\alpha^{(1)} = \int \bar{\Phi}(p)\Bigg\{\left(1+\frac{\gamma^{(2)}L^{(2)}}{m}\right)^2\left(1+\frac{\gamma^{(3)}L^{(3)}}{m}\right)^2 A_\nu(k)\times \\ \times(1+\gamma^{(1)}L^{(1)}/m+\gamma^{(1)}k/2m)\,\gamma_\nu^{(1)}(1+\gamma^{(1)}L^{(1)}/m-\gamma^{(1)}k/2m)\Big\}\,\Phi(p')\times \\ \times\,\delta(p'-p+k)\,\delta[L+k/2-p]\,\delta[L-k/2-p']\,\delta[(L^{(2)}-p/3)p]\times \\ \times\,\delta[(L^{(3)}-p/3)p]\,\delta[(L^{(2)}-p'/3)p']\,\delta[(L^{(3)}-p'/3)p']\,dL^{(1)}dL^{(2)}dL^{(3)}dk.\end{aligned} \tag{17}$$

Let us decompose the vector quantity $\gamma_\nu^{(1)}$ into projections on appropriately chosen axes:

$$\gamma_\nu^{(1)} = L_\nu^{(1)}\frac{(\gamma^{(1)}L^{(1)})}{L^{(1)2}} - k_\nu\frac{(\gamma^{(1)}k)}{k^2} + e_\nu^\perp(\gamma e^\perp). \tag{18}$$

Here $e_\nu^\perp$ is a unit vector lying in the hyperplane orthogonal to $L^{(1)}$ and k.

As can now be verified,

$$(\gamma L^{(1)})\gamma_\nu^{(1)}(\gamma k) - (\gamma k)\gamma_\nu^{(1)}(\gamma L^{(1)}) = (\gamma L^{(1)})\gamma_\nu^{\perp(1)}(\gamma k) - (\gamma k)\gamma_\nu^{\perp(1)}(\gamma L^{(1)}) =$$

$$= 2i\sum_{\mu\nu\rho}\varepsilon_{\nu\mu\rho\sigma}L_\mu^{(1)}k_\rho\gamma_\sigma\gamma_5. \tag{19}$$

From this we obtain an explicit expression for the quantity $\overline{\Phi}\gamma_\nu^{(1)}\Phi$, noting that, with allowance for (10), we have

$$2(m^2-k^2/4)\overline{\Phi}\gamma_\nu^{(1)}\Phi = 2m\overline{\Phi}L_\nu^{(1)}\Phi + \frac{1}{2}\overline{\Phi}[(\gamma L^{(1)})\gamma_\nu^{(1)}(\gamma k) - (\gamma k)\gamma_\nu^{(1)}(\gamma L^{(1)})]\Phi. \tag{20}$$

Carrying out analogous calculations, we can also find the quantities

$$\overline{\Phi}\{(\gamma k)\gamma_\nu^{(1)} - \gamma_\nu^{(1)}(\gamma k)\}\Phi, \qquad \Phi\{(\gamma L^{(1)})\gamma_\nu^{(1)} + \gamma_\nu^{(1)}(\gamma L^{(1)})\}\Phi. \tag{21}$$

Collecting the results (19), (20), and (21), we finally obtain

$$\int \overline{\widetilde{\Psi}}_m \widetilde{\Psi}\, d\tau \Big/ \int \overline{\widetilde{\Psi}} \widetilde{\Psi}\, d\tau = \sum_{(i)} F(k^2) \left\{ \overline{\Phi}\Phi \,(AL^{(i)}) + \frac{iA_\nu}{2m} \Phi \sum_{\mu\nu\rho} \varepsilon_{\nu\mu\rho\sigma} L_\mu^{(i)} k_\rho \gamma_\sigma \gamma_5 \overline{\Phi} \right\} \Delta d\tau. \tag{22}$$

Here the necessary δ-functions written earlier are denoted by Δ, and the common form factor $F(k^2)$ for our model has the form $F(k^2) = m/C[m^2 - k^2/4]$.

As can be verified directly, the relation between the electromagnetic form factors of baryons following from (22) coincides with the experimentally established relations (1). This, of course, follows from the correct form of the second term in the integrand in (22), which we have already discussed above.

Thus, in summary, one may say that, starting from the model of quasi-independent quarks with switched-off interaction, we have been able to obtain a form of the electromagnetic current of baryons which agrees better than the existing ones with the experimental data and which was earlier proposed, without proof, as a hypothesis.

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Note: Figure translations are in progress. See original paper for figures.