THE FUNDAMENTAL MATRIX OF SOLUTIONS OF A SYSTEM OF THE SPECTRAL PROBLEM OF HEAT AND MASS TRANSFER

MATHEMATICS

1968

SovietRxiv

Abstract

Full Text

UDC 517.949.2

MATHEMATICS

M. L. RASULOV

THE FUNDAMENTAL MATRIX OF SOLUTIONS OF A SYSTEM OF THE SPECTRAL PROBLEM OF HEAT AND MASS TRANSFER

(Presented by Academician I. N. Vekua on 11 IX 1967)

It is known that the system of differential equations of heat and mass transfer under molecular and molar transfer of energy and matter in the case of constant coefficients has the form ([1], Ch. VI)

$$\partial v/\partial t = A\Delta v, \tag{1}$$

where A is a square matrix of the third order with constant elements a_{ij} $(i, j = 1, 2, 3)$, and Δ is the three-dimensional Laplace operator with respect to the point $x = (x_1, x_2, x_3)$.

In the note [2] an application is given of the contour-integral method to the solution of a mixed problem for a parabolic system of the form (1) in the case when A is a square matrix of the second order. As is seen from the results of the note [2], the application of the contour-integral method to the solution of a mixed problem for the parabolic system (1) is essentially connected with the construction of the fundamental matrix of solutions of the system of the corresponding spectral problem in finite form.

In this connection, in the present note the fundamental matrix of solutions is given for the system

$$A\Delta u - \lambda^2 u = 0 \tag{2}$$

under the assumption of parabolicity of the system (1).

Let

$$\delta(\mu) = \det(A + \mu E), \qquad \delta = \delta(0),$$

where E is the identity matrix of the third order. Denote by δ_{ks} the algebraic complement of the element (k, s) in the determinant δ.

By virtue of the parabolicity of the system (1), ν —the roots of the equation

$$\nu^3 + (a_{11} + a_{22} + a_{33})\nu^2 + (\delta_{11} + \delta_{22} + \delta_{33})\nu + \delta = 0 \tag{3}$$

have negative imaginary parts.

Theorem 1. *If all roots ν_1, ν_2, ν_3 of equation (3) are distinct, then for the elements $P_{mn}(x, \lambda)$ of the fundamental matrix $P(x, \lambda)$ of solutions of system (2) the following formulas hold:*

$$P_{mn}(x, \lambda) = -\frac{1}{4\pi|x|} \sum_{k=1}^{3} \frac{q_{nm}^{(k)}}{\nu_k} \exp\left[-\lambda \frac{|x|}{\sqrt{-\nu_k}}\right], \tag{4}$$

where $|x|$ is the length of the vector x,

$$q_{nm}^{(k)} = \frac{-a_{mn}\nu_k + \delta_{nm}}{\varepsilon_k} \qquad (n \neq m),$$

$$\varepsilon_1 = (\nu_1 - \nu_2)(\nu_1 - \nu_3), \qquad \varepsilon_2 = (\nu_2 - \nu_1)(\nu_2 - \nu_3),$$

$$\varepsilon_3 = (\nu_3 - \nu_1)(\nu_3 - \nu_2),$$

$$q_{11}^{(k)} = \frac{\nu_k^2 + (a_{22} + a_{33})\nu_k + \delta_{11}}{\varepsilon_k},$$

$$q_{22}^{(k)} = \frac{\nu_k^2 + (a_{11} + a_{33})\nu_k + \delta_{22}}{\varepsilon_k}, \qquad q_{33}^{(k)} = \frac{\nu_k^2 + (a_{11} + a_{22})\nu_k + \delta_{33}}{\varepsilon_k}.$$

Theorem 2. If equation (3) has two distinct roots ν_1, ν_2, where ν_1 is a root of multiplicity two, then for the elements $P_{mn}(x, \lambda)$ of the fundamental matrix $P(x, \lambda)$ of solutions of system (2), the following formulas hold:

$$\begin{aligned} P_{mn}(x, \lambda) = -\frac{1}{8\pi} \Bigg\{ &\frac{q_{mn}^{(1)}}{\lambda(-\nu_1)^{3/2}} \Delta \exp\left[-\lambda \frac{|x|}{\sqrt{-\nu_1}}\right] \\ &+ \frac{2q_{mn}^{(2)}}{\nu_1 |x|} \exp\left[-\lambda \frac{|x|}{\sqrt{-\nu_1}}\right] + \frac{2q_{mn}^{(3)}}{\nu_2 |x|} \exp\left[-\lambda \frac{|x|}{\sqrt{-\nu_2}}\right] \Bigg\}, \end{aligned} \tag{5}$$

$$(n, m = 1, 2, 3),$$

where

$$q_{mn}^{(1)} = \frac{-a_{mn}\nu_1 + \delta_{mn}}{\nu_1 - \nu_2} \qquad (\text{for } n \neq m),$$

$$-q_{mn}^{(2)} = q_{mn}^{(3)} = \frac{-a_{mn}\nu_2 + \delta_{mn}}{(\nu_1 - \nu_2)^2} \qquad (\text{for } n \neq m;\ n, m = 1, 2, 3),$$

$$q_{11}^{(1)} = \frac{\nu_1^2 + (a_{22} + a_{33})\nu_1 + \delta_{11}}{\nu_1 - \nu_2}, \qquad q_{11}^{(2)} = \frac{\nu_1^2 - [2\nu_1 + (a_{22} + a_{33})]\nu_2 - \delta_{11}}{(\nu_1 - \nu_2)^2},$$

$$q_{11}^{(3)} = \frac{\nu_2^2 + (a_{22} + a_{33})\nu_2 + \delta_{22}}{(\nu_1 - \nu_2)^2}, \qquad q_{22}^{(1)} = \frac{\nu_1^2 + (a_{11} + a_{33})\nu_1 + \delta_{22}}{\nu_1 - \nu_2},$$

$$q_{22}^{(2)} = \frac{\nu_1^2 - [2\nu_1 + (a_{11} + a_{33})]\nu_2 - \delta_{22}}{(\nu_1 - \nu_2)^2}, \qquad q_{22}^{(3)} = \frac{\nu_2^2 + (a_{11} + a_{33})\nu_2 + \delta_{22}}{(\nu_1 - \nu_2)^2},$$

$$q_{33}^{(1)} = \frac{\nu_1^2 + (a_{11} + a_{22})\nu_1 + \delta_{33}}{\nu_1 - \nu_2},$$

$$q_{33}^{(2)} = \frac{\nu_1^2 - [2\nu_1 + (a_{11} + a_{22})]\nu_2 - \delta_{33}}{(\nu_1 - \nu_2)^2}, \qquad q_{33}^{(3)} = \frac{\nu_2^2 + (a_{11} + a_{22})\nu_2 + \delta_{33}}{(\nu_1 - \nu_2)^2}.$$

Theorem 3. If equation (3) has only one root ν of multiplicity three, then for the elements $P_{mn}(x, \lambda)$ of the fundamental matrix $P(x, \lambda)$ of solutions of system (2), the following formulas hold:

$$P_{mn}(x,\lambda) = \frac{1}{8\pi}\left\{ \frac{q_{mn}^{(1)}}{4\lambda^3(-\nu)^{3/2}} \Delta^2 (1 + \lambda|x|) \exp\left[-\lambda \frac{|x|}{\sqrt{-\nu}}\right] - \frac{q_{mn}^{(2)}}{\lambda(-\nu)^{1/2}} \Delta \exp\left[-\lambda \frac{|x|}{\sqrt{-\nu}}\right] - \frac{2q_{mn}^{(3)}}{\nu|x|} \exp\left[-\lambda \frac{|x|}{\sqrt{-\nu}}\right] \right\}, \tag{6}$$

where

$$q_{mn}^{(1)} = -a_{mn}\nu + \delta_{mn}, \qquad q_{mn}^{(2)} = -a_{mn}, \qquad q_{mn}^{(3)} = 0 \quad \text{for } n \neq m,$$

$$q_{mm}^{(3)} = 0 \qquad (m = 1, 2, 3),$$

$$q_{11}^{(1)} = \nu^2 + (a_{22} + a_{33})\nu + \delta_{11}, \qquad q_{11}^{(2)} = 2\nu + (a_{22} + a_{33}),$$

$$q_{22}^{(1)} = \nu^2 + (a_{11} + a_{33})\nu + \delta_{22}, \qquad q_{22}^{(2)} = 2\nu + (a_{11} + a_{33}),$$

$$q_{33}^{(1)} = \nu^2 + (a_{11} + a_{22})\nu + \delta_{33}, \qquad q_{33}^{(2)} = 2\nu + (a_{11} + a_{22}).$$

With the aid of the fundamental matrix $P(x, \lambda)$ of solutions of system (2), exactly as was done in (2), one can prove the validity of Theorems 1, 2, 3 of note (2) for system (1).

Azerbaijan State University
named after S. M. Kirov

Received
31 VII 1967

CITED LITERATURE

1. A. V. Lykov, Yu. A. Mikhailov, *Theory of the Transfer of Energy and Matter*, Minsk, 1959.

2. M. L. Rasudov, DAN, 177, No. 6 (1967).

Note: Figure translations are in progress. See original paper for figures.