CONDITION FOR THE EXCITATION OF SELF-OSCILLATIONS IN NONLINEAR DIRECT-CURRENT CIRCUITS

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Abstract

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CONDITION FOR THE EXCITATION OF SELF-OSCILLATIONS IN NONLINEAR DIRECT-CURRENT CIRCUITS

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Electrical circuits containing nonlinear reactive elements—capacitive and (or) inductive—find numerous applications in modern technology. On their basis it is possible to construct almost all functional elements needed for the implementation of various systems of automation and computer technology. New types and applications of "magnetic" and "dielectric" elements, based on the use of the nonlinear properties of ferromagnets and ferroelectrics, are developing intensively.

At the same time, there exist two important functions that cannot be performed by means of nonlinear reactive elements: with the aid of nonlinear inductances and (or) capacitances it is (a) impossible to rectify alternating current and (b) impossible to convert direct current into alternating current.

Whereas the impossibility of rectifying alternating current or voltage by means of nonlinear reactive elements is easily proved, is well known, and usually raises no doubts, the impossibility of converting direct current into alternating current by means of nonlinear reactive elements is less well known, and sometimes researchers and inventors make unsuccessful attempts to create magnetic and dielectric alternating-current generators operating from a direct-current source. The impossibility of creating such generators was apparently first proved in ([1]). It can also be obtained from the general theory of nonlinear circuits developed in ([3]). Below we give a somewhat different proof of this impossibility, free of some restrictions present in ([1]).

Let us have an arbitrary electrical circuit containing m_Γ sources of constant voltage, m_L inductors, m_C capacitors, and m_R resistors. The active resistances of the inductor windings may also be attributed to the resistors. The inductances of the inductors, the capacitances of the capacitors, and the resistances of the resistors may be nonlinear. According to the law of conservation of energy we have ([2])

$$\sum ui = 0, \tag{1}$$

where u and i are respectively the voltage and current of one of the circuit elements, and the summation is carried out over all elements. If $ui > 0$, then the given element consumes energy; if $ui < 0$, then it is a source of energy.

For the power source $u = -E$, for an inductive element $u = d\psi/dt$, and for a capacitive element $i = dq/dt$. Therefore (1) can be written in the form

$$\sum^{m_\Gamma} Ei = \sum^{m_L} i\,\frac{d\psi}{dt} + \sum^{m_C} u\,\frac{dq}{dt} + \sum^{m_R} ui. \tag{2}$$

Suppose that stable self-oscillations with period T arise in the circuit. Then each u and i can be represented as the sum of a constant (U_0, I_0) and a periodic $(u_\sim, i_\sim)$ component:

$$u = U_0 + u_\sim; \qquad i = I_0 + i_\sim. \tag{3}$$

For active resistances one may take

$$u_\sim = R_\text{d}(i) i_\sim, \tag{4}$$

where, in the general case, $R_\text{d}(i)$ is a certain function of the current. Obviously, for linear resistances $R_\text{d}(i) = R = \text{const}$. For small oscillation amplitudes ($u_\sim \ll U_0$ and $i_\sim \ll I_0$), for nonlinear resistances,

$$u_\sim = du/di\big|_{i=I_0} i_\sim = R_\text{d}(I_0) i_\sim, \tag{5}$$

i.e., in this case $R_\text{d}(i)$ coincides with the generally accepted differential resistance du/di. In the general case, however, they of course differ.

Let us divide all nonlinear active resistances of the circuit under consideration into two groups: (a) positive ones, for which, as for linear resistances, $R_\text{d}(i) > 0$, and (b) negative ones, for which $R_\text{d}(i) < 0$ over the entire range of variation of their current in the given circuit. Obviously, $R_\text{d}(i) < 0$ only in the case where $du/di < 0$.

Let us find the energy released in the circuit during one period T of the established oscillations:

$$W = \int_0^T \sum^{m_\Gamma} Ei\,dt = T \sum^{m_\Gamma} EI_0, \tag{6}$$

since

$$\int_0^T i_\sim \, dt = 0. \tag{7}$$

In the established regime,

$$\int_0^T i \, d\psi = A_L \geq 0, \tag{8}$$

where A_L is the loss due to remagnetization of the core of the choke when its flux linkage changes cyclically from $\psi(0)$ to $\psi(T) = \psi(0)$. In the absence of a core, $A_L = 0$.

In the case of a multiwinding choke (transformer),

$$\int_0^T \sum_k i_k \, d\psi_k = A_L, \tag{9}$$

where i_k and ψ_k are, respectively, the current and flux linkage of the k-th winding.

Thus,

$$\int_0^T \sum^{m_L} i \, d\psi = \sum^{m_L} A_L = \sigma_L \geq 0. \tag{10}$$

Similarly,

$$\int_0^T \sum^{m_C} u \, dq = \sum^{m_C} A_C = \sigma_C \geq 0, \tag{11}$$

where A_C is the loss in the capacitor when its charge changes cyclically from $q(0)$ to $q(T) = q(0)$.

For an active resistance, taking (7) into account, we have

$$\int_0^T ui \, dt = TU_0 I_0 + \int_0^T u_\sim i_\sim \, dt. \tag{12}$$

In the case of a linear or positive nonlinear resistance,

$$u_\sim i_\sim = R_\text{d}(i) i_\sim^2 \geqslant 0,$$

and for a negative resistance

$$u_\sim i_\sim = R_\text{d}(i) i_\sim^2 \leqslant 0.$$

Therefore formula (12) can be written in the form

$$\int_0^T ui\,dt = TU_0 I_0 + \widetilde{A}_R \operatorname{sign} R_\text{d}(i), \tag{13}$$

where

$$\widetilde{A}_R = \left| \int_0^T R_\text{d}(i) i^2\,dt \right| > 0.$$

Thus we obtain

$$\sum^{m_\Gamma} EI_0 = \frac{1}{T}(\sigma_L + \sigma_C) + \sum^{m_R} \left[U_0 I_0 + P_\sim \operatorname{sign} R_\text{d}(i)\right], \tag{14}$$

where $P_\sim = \widetilde{A}_R / T$ is the mean power of the alternating current.

In the absence of oscillations $\sigma_L = 0$, $\sigma_C = 0$, $P_\sim = 0$, and

$$\sum^{m_\Gamma} EI_0 = \sum^{m_R} U_0 I_0. \tag{15}$$

This regime occurs, in particular, if all inductances are short-circuited and all capacitors are disconnected. It is obvious that, when reactive elements are connected, self-oscillations can arise only under the condition that

$$\sum^{m_R} P_\sim \operatorname{sign} R_\text{d}(i) + \frac{1}{T}(\sigma_L + \sigma_C) < 0. \tag{16}$$

Since $\sigma_L + \sigma_C \geqslant 0$ and $P_\sim > 0$, a necessary condition for the existence of undamped self-oscillations is the presence in the circuit of at least one negative nonlinear resistance $R_\text{d}(i) < 0$.

Thus, the excitation of self-oscillations is impossible in circuits with direct-current sources that contain arbitrary nonlinear inductances, capacitances, and resistances, if among the latter there is no negative resistance.

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Note: Figure translations are in progress. See original paper for figures.