ALGEBRA OF MESON CURRENTS

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Abstract

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ALGEBRA OF MESON CURRENTS

IN THE MODEL OF FREE EFFECTIVE QUARKS

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1°. Recently, in the theory of elementary particles, an approach known as current algebra has been widely used; with its help a number of important relations between physically observable quantities have been obtained. However, the relations of current algebra have not yet been proved in the general case and are, generally speaking, a certain postulate, whose validity is checked only by the results obtained with its aid. It is therefore of interest to test current algebra within the framework of definite models that can be successfully applied in the description of elementary particles. Such models are quark models, in which hadrons are regarded as composite particles. In the present communication we shall show that, within the framework of the model of free effective quarks [1−3], it is possible to prove the validity of the relations of the algebra of meson currents to any order of the expansion in the inverse meson mass.

2°. In the effective-quark model a meson is regarded as a strongly bound state of a quark and an antiquark with very large masses; moreover, these masses are essentially compensated by a strong intrahadron interaction, which may be interpreted as a certain self-consistent field. The excess of the quark masses, exactly equal to the mass of the hadron, is assumed to be connected with certain effective formations in the hadron, moving in a quasi-independent manner in this field. The interaction between quasi-independent (effective) quarks is very small and may be excluded from consideration altogether, which corresponds to the model of free effective quarks that we shall use.

Consider a meson consisting of an effective quark with mass m_1 and an anti-quark with mass m_2, having momenta respectively $l^{(1)}$ and $l^{(2)}$. The meson has momenta p and p' in the initial and final states, defined respectively as

$$p = l^{(1)} + l^{(2)}, \qquad p' = {l'}^{(1)} + {l'}^{(2)}. \tag{1}$$

The relative momenta inside the meson are defined by the formulas

$$v = l^{(1)} - l^{(2)}, \qquad v' = l'^{(1)} - l'^{(2)}. \tag{2}$$

In this model the nonrelativistic meson current is written in the form

$$I_{a,\beta}(0,\mathbf{x}) = \delta(\mathbf{x} - \mathbf{x}^{(1)})\Gamma^{(1)}_{a,\beta} + \delta(\mathbf{x} - \mathbf{x}^{(2)})\Gamma^{(2)}_{a,\beta}, \tag{3}$$

where $\mathbf{x}$ is the coordinate of the meson, $\mathbf{x}^{(1),(2)}$ are the coordinates of the quark and antiquark, respectively, and $\Gamma^{(i)}_{a,\beta}$ are operators of the form

$$\lambda^{(i)}_b \gamma^{(i)}_\beta (\delta)^{(j)}, \qquad a = 0, \dots, 8; \quad \beta = 0, \dots, 3; \quad j \neq i, \tag{4}$$

where $\lambda^{(i)}_a$ are the Gell-Mann matrices acting respectively on the quark or antiquark, and γ_β are any of the Dirac matrices.

From translational invariance it follows that

$$(\mathbf{p}', \vec{v}' \mid I_a(0,\mathbf{x}) \mid \mathbf{p}, \vec{v}) = \exp[i(\mathbf{p} - \mathbf{p}')\mathbf{x}](\mathbf{p}', \vec{v}' \mid I_a(0) \mid \mathbf{p}, \vec{v}). \tag{5}$$

The vertices introduced in (5) are defined in the following way in terms of the wave functions Ψ (the internal wave functions of the meson):

$$(\mathbf{p}', \vec{v}' \mid F_{a,\beta}(\mathbf{k}) \mid \mathbf{p}, \vec{v}) = \int \Psi^+_{\mathbf{p}',\vec{v}'}(\mathbf{x}^{(1)}, \mathbf{x}^{(2)}) F_{a,\beta}(\mathbf{k}) \Psi_{\mathbf{p},\vec{v}}(\mathbf{x}^{(1)}, \mathbf{x}^{(2)})\, d\mathbf{x}^{(1)}\, d\mathbf{x}^{(2)}, \tag{6}$$

where

$$F_{a,\beta}(\mathbf{k}) \equiv \int I_{a,\beta}(0,\mathbf{x}) \exp(i\mathbf{k}\mathbf{x})\, d\mathbf{x} = \sum_{(i)} \exp(i\mathbf{k}\mathbf{x}^{(i)}) \Gamma^{(i)}_{a,\beta}. \tag{7}$$

Next we introduce a wave function φ, characterizing the individual momenta of the quark and antiquark inside the meson and related to the internal wave function of the meson Ψ by the formula

$$\Psi_{\mathbf{p},\vec{v}}(\mathbf{x}^{(1)}, \mathbf{x}^{(2)}) = \frac{1}{(2\pi)^3} \int \exp[i(\mathbf{l}^{(1)}\mathbf{x}^{(1)} + \mathbf{l}^{(2)}\mathbf{x}^{(2)})] \times$$

$$\times\, \delta(\mathbf{l}^{(1)} + \mathbf{l}^{(2)} - \mathbf{p})\, \varphi_{\vec{v}}(\mathbf{l}^{(1)}, \mathbf{l}^{(2)})\, d\mathbf{l}^{(1)}\, d\mathbf{l}^{(2)}. \tag{8}$$

Using (6)–(8), we obtain an expression for the nonrelativistic matrix element of the meson current

$$(\mathbf{p}', \vec{v}' \mid F_{a,\beta}(\mathbf{k}) \mid \mathbf{p}, \vec{v}) = (2\pi)^3 \delta(\mathbf{p} - \mathbf{p}' + \mathbf{k})(\mathbf{p}', \vec{v}' \mid I_{a,\beta}(0) \mid \mathbf{p}, \vec{v}), \qquad (9)$$

where

$$(\mathbf{p}', \vec{v}' \mid I_{a,\beta}(0) \mid \mathbf{p}, \vec{v}) = \frac{1}{(2\pi)^3} \delta(\mathbf{l}'^{(1)} + \mathbf{l}'^{(2)} - \mathbf{p}') \delta(\mathbf{l}^{(1)} + \mathbf{l}^{(2)} - \mathbf{p}) \times$$

$$\times \varphi^+_{\vec{v}'}(\mathbf{l}'^{(1)}, \mathbf{l}'^{(2)}) \left\{ \delta(\mathbf{l}^{(2)} - \mathbf{l}'^{(2)}) \Gamma^{(1)}_{a,\beta} + \delta(\mathbf{l}^{(1)} - \mathbf{l}'^{(1)}) \Gamma^{(2)}_{a,\beta} \right\} \times$$

$$\times \varphi_{\vec{v}}(\mathbf{l}^{(1)}, \mathbf{l}^{(2)}) \, d\mathbf{l}^{(1)} \, d\mathbf{l}^{(2)} \, d\mathbf{l}'^{(1)} \, d\mathbf{l}'^{(2)}. \qquad (10)$$

3°. Relativistically generalizing this expression in accordance with the scheme described in (2), and using the Markov-Yukawa conditions (2,3) in the form (for the term containing, for example, $\Gamma^{(1)}$)

$$\delta(l^{(1)2} - m_1^2) \, \delta({l'}^{(2)2} - m_2^2) \, \delta[pv - \Delta m^2] \, \delta[p'v' - {\Delta m'}^2], \qquad (11)$$

where

$$\Delta m^2 = m_2^2 - m_1^2; \qquad {\Delta m'}^2 = m_2^2 - m_1'^2, \qquad (12)$$

we define the relativistic matrix element of the "internal" meson current as follows:

$$\langle p', v' \mid I_{a,\beta}(0) \mid p, v \rangle = (2\pi)^{-3} \int \delta({l'}^{(1)} + {l'}^{(2)} - p') \delta(l^{(1)} + l^{(2)} - p) \times$$

$$\times \overline{\varphi}_{v'}({l'}^{(1)}, {l'}^{(2)}) \left\{ \delta({l'}^{(2)2} - m_2^2)(l^{(2)} + {l'}^{(2)})_0 \delta(\mathbf{l}^{(2)} - \mathbf{l}'^{(2)}) \Gamma^{(1)}_{a,\beta} C_1 \delta(l^{(1)2} - m_1^2) + \right.$$

$$\left. + \delta({l'}^{(1)2} - m_1^{(2)})(l^{(1)} + {l'}^{(1)})_0 \delta(\mathbf{l}^{(1)} - \mathbf{l}'^{(1)}) \Gamma^{(2)}_{a,\beta} C_2 \delta(l^{(2)2} - m_2^2) \right\} \times$$

$$\times \varphi_v(l^{(1)}, l^{(2)}) \prod_{i=1}^{2} dl^{(i)} \, d{l'}^{(i)}. \qquad (13)$$

Here $\overline{\varphi}$ is a Dirac-contracted quantity, and the angular brackets $\langle p, v \mid$ are normalized to the product of two relativistically covariant quantities of the form

$$\frac{1}{2l_0^{(i)}}\,\delta(\mathbf{l}^{(i)}-\mathbf{l}'^{(i)}) \tag{14}$$

in contrast to round brackets, normalized to three-dimensional δ-functions ([4]).

The wave function φ, introduced in (8), has the form

$$\varphi^{\alpha,\beta}(\mathbf{p},\mathbf{v}) \equiv |p,\mathbf{v}\rangle = p_0\delta\big[l^{(1)}-l^{(2)}-\mathbf{v}\big]\;U_\alpha(l^{(1)})\,\bar{U}_\beta(l^{(2)})\,R_1^{-1}R_2^{-1}, \tag{15}$$

where $U_\alpha(l^{(i)})$ are solutions of the free Dirac equation, and R are spatial rotations corresponding to the Lorentz transformation Λ from the meson rest system to an arbitrary system $(p,0)$, and are defined by the expression

$$R = L^{-1}_{n^0\to\Lambda p/m}\Lambda L_{n^0\to p/m}. \tag{16}$$

Here n^0 is the time unit vector in the meson rest system.

Using these formulas, we obtain for the relativistic matrix element of the "internal" meson current the expressions

$$\begin{aligned}\langle p',\mathbf{v}',\alpha',\beta'|F_a(k)|p,\mathbf{v},\alpha,\beta\rangle = \frac{p_0p_0'R_2'R_1'}{16}\Bigg\{&\delta_{\beta\beta'}\bar{u}_{\alpha'}\left(\frac{p'+v'}{2}\right)\Gamma_a^{(1)}C_1u_\alpha\left(\frac{p+v}{2}\right)\delta(\mathbf{p}-\mathbf{v}-\mathbf{p}'-\mathbf{v}')\\&\times\delta(\mathbf{v}-\mathbf{v}'+\mathbf{k})\frac{p_0-v_0+p_0'-v_0'}{(p_0'-v_0')(p_0+v_0)}\\&+\delta_{\alpha\alpha'}U_{\beta'}\left(\frac{p'-v'}{2}\right)\Gamma_a^{(2)}C_2\bar{U}_\beta\left(\frac{p-v}{2}\right)\delta(\mathbf{p}+\mathbf{v}-\mathbf{p}'-\mathbf{v}')\\&\times\delta(\mathbf{v}'-\mathbf{v}+\mathbf{k})\frac{p_0+v_0+p_0'+v_0'}{(p_0'+v_0')(p_0-v_0)}\Bigg\}R_1^{-1}R_2^{-1},\end{aligned} \tag{17}$$

and, passing to round brackets with the aid of the additional factor

$$\left[\frac{1}{4}(p_0^2-v_0^2)\right]^{-1}, \tag{18}$$

we obtain instead of (17) the formula

$$(p',\mathbf{v}',\alpha',\beta'|F_a(k)|p,\mathbf{v},\alpha,\beta) = \frac{p_0 p_0' R_2' R_1'}{2(p_0+v_0)(p_0-v_0)} \times \left\{ \frac{\delta_{\beta\beta'}\bar{u}_{\alpha'}((p'+v')/2)\Gamma_a^{(1)} C_1 U_\alpha((p+v)/2)\delta(\mathbf{p}-\mathbf{v}-\mathbf{p}'+\mathbf{v}')\delta(\mathbf{v}-\mathbf{v}'+\mathbf{k})}{\sqrt{(p_0+v_0)(p_0'+v_0')}} + \frac{\delta_{\alpha\alpha'} U_{\beta'}((p'-v')/2)\Gamma_a^{(2)} C_2 \bar{U}_\beta((p-v)/2)\delta(\mathbf{p}+\mathbf{v}-\mathbf{p}'-\mathbf{v}')\delta(\mathbf{v}'-\mathbf{v}+\mathbf{k})}{\sqrt{(p_0-v_0)(p_0'-v_0')}} \right\} \tag{19}$$

Now specifying the reference system $(p,\mathbf{v})$, and assuming that

$$p_z \to \infty, \qquad p_z' \to \infty, \tag{20}$$

while p^2 and ${p'}^2$ remain finite quantities, we find from (19)

$$(p',\mathbf{v}',\alpha',\beta'|F_a(k)|p,\mathbf{v},\alpha,\beta) = R_2' R_1' \delta(\mathbf{p}+\mathbf{k}-\mathbf{p}') \times \left\{ \delta_{\beta\beta'}\delta(\mathbf{v}-\mathbf{v}'+\mathbf{k})\bar{\chi}_{\alpha'}\Lambda_a^{(1)}\chi_\alpha + \delta_{\alpha\alpha'}\delta(\mathbf{v}-\mathbf{v}'-\mathbf{k})\chi_{\beta'}\Lambda_a^{(2)}\bar{\chi}_\beta \right\} R_1^{-1} R_2^{-1}. \tag{21}$$

Here the operators $\chi_a^{(i)}$ are defined by formulas of the form

$$\Lambda_a^{(i)} = \begin{cases} \lambda_a^{(i)}, \\ \lambda_a^{(i)}\sigma_3^{(i)}(-1)^{i+1}, \end{cases} \tag{22}$$

where the upper value applies to the vector current, and the lower to the axial current. By direct verification it is now not difficult to see that the commutation relations for the matrix elements (21) have the form

$$\begin{aligned}&[(p',\mathbf{v}',\alpha',\beta'|F_a(k)|p'',\mathbf{v}'',\alpha'',\beta''),(p'',\mathbf{v}'',\alpha''\beta''|F_b(k')|p,\mathbf{v},\alpha,\beta)] \\ &\quad = if_{abc}(p',\mathbf{v}',\alpha',\beta'|F_c(k+k')|p,\mathbf{v},\alpha,\beta),\end{aligned} \tag{23}$$

where

$$\lambda_a^{(i)}\lambda_b^{(i)} - \lambda_b^{(i)}\lambda_a^{(i)} \equiv if_{abc}\lambda_c^{(i)}, \tag{24}$$

and the integration on the left-hand side of (23) is over the spatial parts $\mathbf{p}''$ and $\mathbf{v}''$, and the summation is over α'' and β''.

4°. Let us now introduce the true wave function of the meson in an arbitrary reference frame in the form

$$\Psi(p,l,W) \equiv |p,l,W\rangle = \int |p,\nu\rangle\rangle\, \delta[p\nu - \Delta m^2]\Psi_{W,l}(\nu)\, d\nu, \tag{25}$$

where here the δ-function supplements the Markov–Yukawa conditions, and the function $\Psi_{W,l}(\nu)$ is defined as

$$\Psi_{W,l}(q) = f(W)\delta\left(W^2 + q^2 - \frac{m_1^2 + m_2^2}{2}\right)\varphi_l(e). \tag{26}$$

Here $l = I, I_z$,

$$e = q/\sqrt{q^2} \equiv (\varphi, \theta), \tag{27}$$

$$f(W) = \sqrt{\frac{2W}{\sqrt{W^2 - \Sigma m^2/2 + (\Delta m^2)^2/16W^2}}}, \tag{28}$$

which ensures the normalization relations

$$\int_0^\infty \Psi^+_{W',l'}(q)\Psi_{W,l}(q)\, dq = \delta(W - W')\delta_{ll'},$$

$$\sum_l \int_0^\infty \Psi^+_{W,l}(q)\Psi_{W,l}(q')\, dW = \delta(q - q'). \tag{29}$$

The wave functions $|p,l,W\rangle$ commute with $\hat{I}$ and $\hat{I}_z$ and thus describe the state of a meson with a definite angular momentum and its projection.

In passing from $|p,\nu\rangle\rangle$ to $|p,l,W\rangle$ in the intermediate states of relations (23), there will now be added new quantities caused by the need to integrate over the additional variables of the factors contained in (25), apart from the internal meson wave function, and given by the expression

$$\sum_l \int_0^\infty dW\, \Psi_{W,l}(\nu)\Psi^+_{W,l}(\nu')\, \delta(p\nu - \Delta m^2)\delta(p\nu' - \Delta m^2) =$$

$$= \frac{p_0\delta(\nu - \nu')\delta(\nu_0 - \vec{\nu}\vec{p}/p_0 - \Delta m^2/p_0)}{8\sqrt{p^2(p_0^2 - \nu_0^2)}}. \tag{30}$$

Thus, in its essential part this factor provides the transition from envelopes of the form $|p,\nu\rangle$ to the true meson wave functions, normalized in a Lorentz-covariant manner. Consequently, relations of the form

$$[\langle p', l', W'|F_a(k)|p'', l'', W''\rangle\langle p'', l'', W''|F_b(k')|p, l, W\rangle] =$$

$$= if_{abc}\langle p', l', W'|F_c(k+k')|p, l, W\rangle. \quad (31)$$

Thus the validity of the current-algebra relations for a meson with the interaction between effective quarks switched off has been proved, independently of the assumption that the matrix elements used can be expanded in inverse meson masses.

The same proves the validity of the current algebra for mesons in the general case up to the second order (inclusive) of the expansion in the inverse meson mass, since the interaction potential between effective quarks enters the consideration only starting with the third order of this expansion ([1]).

An analogous result can also be obtained for baryons ([3]).

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Note: Figure translations are in progress. See original paper for figures.