USE OF THERMAL DATA TO REDUCE THE NONUNIQUENESS OF INVERSE PROBLEMS OF SEISMOLOGY

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Abstract

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USE OF THERMAL DATA TO REDUCE THE NONUNIQUENESS OF INVERSE PROBLEMS OF SEISMOLOGY

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With the existing accuracy of seismic observations, their interpretation leads to considerable nonuniqueness in estimating the velocity section of the Earth. The same seismic data may be brought into correspondence with many velocity sections that differ substantially from one another in geophysical significance or in the dynamics of the wave pattern associated with them ([1−3]). The nonuniqueness of the solution of the inverse problem of seismology concerning the determination of the velocity section can be partly reduced by jointly interpreting seismic observations of different classes, in particular body and surface waves.

It was precisely in this way that the inverse problem was solved by V. P. Valyus, A. L. Levshin, and T. M. Sabitova ([2]), who determined the velocity section of the Earth' s crust in Central Asia along the Andizhan–Dushanbe profile, 400 km long. The hodographs of body waves $t_l(\Delta)$ and $t_s(\Delta)$ were taken from data of the complex seismological expedition of the Institute of Physics of the Earth, Academy of Sciences of the USSR. The travel times of the first arrivals of the l and s waves were used at distances $\Delta = 50, 100, 150, 250, 300$, and 350 km. Many dozens of sections satisfying the observed seismic data were obtained. Some of them are shown in Fig. 1. It is seen that the possible velocity sections occupy a very wide band. The interval of possible positive velocity gradients dv_l/dz in the granitic layer extends from 0 to 0.07 sec^{-1}, and in the basaltic layer from 0 to 0.02 sec^{-1}. The combined interpretation of the observed seismic data is still insufficient for a substantial reduction of nonuniqueness.

The purpose of the present note is to show that the domain of possible solutions of the inverse problem of seismology concerning the deep velocity section can be substantially reduced by bringing in thermal data.

The velocities of seismic waves depend on temperature T and pressure P. Therefore the change of the velocities v_l and v_s with depth z may be written in the form

$$dv_a/dz = (dv_a/dP)_T \, dP/dz + (dv_a/dT)_P \, dT/dz, \qquad a = l,\ s. \tag{1}$$

In the upper layers of the Earth, in particular in the granitic and basaltic layers, the temperature gradient changes especially rapidly with depth, whereas the pressure gradient remains practically constant, $dP/dz = 260$ bar/km. Let us denote $\alpha_l = (dv_l/dP)_T$, $\beta_l = (dv_l/dT)_P$, $\alpha_s = (dv_s/dP)_T$, $\beta_s = (dv_s/dT)_P$.

The velocities of seismic waves increase with pressure and decrease with temperature; therefore $\alpha_l, \alpha_s > 0$, $\beta_l, \beta_s < 0$. Since in the Earth' s interior $dT/dz > \gamma > 0$ (where γ is the adiabatic gradient), expression (1) leads to the condition on the velocity gradient:

$$\frac{1}{\beta_a}\frac{dv_a}{dz} - \frac{\alpha_a}{\beta_a}\frac{dP}{dz} > \gamma, \qquad a = l,\ s. \tag{2}$$

It is possible to analyze the set of velocity sections of the Earth' s crust obtained by methods of theoretical seismology from the standpoint of satisfaction of condition (2). The coefficients $\alpha_l, \alpha_s, \beta_l, \beta_s$ can be deter-

Fig. 1. Set of velocity sections of the Earth' s crust along the Andizhan—Dushanbe profile, satisfying observational seismic data of different classes according to [2]. Sections forbidden from the thermal point of view are accompanied by black circles; z_2, z_3 are parametrized thicknesses of crustal layers.

mined from data on the velocities v_l, v_s, obtained by a number of investigators under laboratory conditions [4, 6]. Their values, recalculated for the corresponding depths of occurrence of the granite, basalt, and dunite layers, are given in Table 1.

Table 1

Rock type	$\alpha_l,\ \frac{\text{km/sec}}{\text{bar}}$	$\beta_l,\ \frac{\text{km/sec}}{^\circ\text{C}}$	Calculated from data
Granite	$4 \cdot 10^{-5}$	$2.5 \cdot 10^{-4}$	(5)
Granite	$1 \cdot 10^{-5}$	$2.0 \cdot 10^{-3}$	(4)
Granite	$(2—7) \cdot 10^{-5}$	–	(6)
Basalt	$1 \cdot 10^{-5}$	$(1—3) \cdot 10^{-4}$	(4)
Dunite	$2 \cdot 10^{-5}$	$(0.7—2.5) \cdot 10^{-3}$	(4, 5)

Fixing α_l, β_l and dP/dz in formula (1), and assigning dv_l/dz from the intervals in Fig. 1 corresponding to the calculations [2], one can determine a graph of the dependence of dT/dz on dv_l/dz. One example of such a dependence for longitudinal seismic waves within the granite layer at different α_l is shown in Fig. 2. From consideration of Fig. 2 it follows that large positive velocity gradients $dv_l/dz > 0.01$ sec^{-1} correspond to negative temperature gradients

Figure 2

Figure 1: Figure 2

dT/dz, which has no physical meaning, since temperature increases with depth. Thus, velocity sections with dv_l/dz in the interval (0.01—0.07) $\sec^{-1}$, which seemed possible from the seismic standpoint, must be rejected.

These sections with forbidden gradients in the Earth' s crust are marked in Fig. 1 by black circles.

Parametrization II from paper (2) was used, under which a weak negative velocity gradient is allowed. From Fig. 1 it is seen that the contour enclosing solutions that satisfy the seismic observations for one class of waves (body waves) occupies a broader subregion (hatched) than the solutions satisfying all the seismic observations used. If, however, the thermal conditions are taken into account, then the subregion of possible solutions is reduced still further. Since one of the varied parameters was the total thickness of the crust, the possible limits of its variation are narrowed. Thus, the use of thermal data does indeed reduce the ambiguity of the solutions. In Fig. 2 the region of the most probable temperature gradients is hatched.

Fig. 2. Relation between the velocity gradient of seismic waves and the temperature gradient dT/dz (deg/km) in the granite layer for values $\alpha_l = 1 \cdot 10^{-5}$ (dashed lines), $\alpha_l = 4 \cdot 10^{-5}$ (solid lines), and values $\beta = 2.0 \cdot 10^{-4}$ (1), $\beta = 2.5 \cdot 10^{-4}$ (2), $\beta = 2 \cdot 10^{-3}$ (3)

A set of velocity sections for the upper mantle can also be analyzed according to the degree of agreement with the required temperature gradients. We used the velocity sections obtained by Keylis-Borok (3) for the territory of North America. The corresponding temperature gradients are calculated in Table 2. It is seen that, for the mantle, at the small velocity gradients adopted, the sections correspond to positive temperature gradients. The magnitude of the latter fully corresponds to real physical conditions in the mantle within the accuracy of the formulation of the problem.

Table 2

Depth, km	dv/dz, 10^{-3} $\sec^{-1}$	dT/dz, °C/km—$\beta_l \times 10^3$ 0.5	dT/dz, °C/km—$\beta_l \times 10^3$ 0.7	dT/dz, °C/km—$\beta_l \times 10^3$ 2.5	Depth, km	dv/dz, 10^{-3} $\sec^{-1}$	$-dT/dz$, °C/km—$\beta_l \times 10^3$ 0.5	$-dT/dz$, °C/km—$\beta_l \times 10^3$ 0.7	$-dT/dz$, °C/km—$\beta_l \times 10^3$ 2.5
Section I (model 705)	**Section I (model 705)**	**Section I (model 705)**	**Section I (model 705)**	**Section I (model 705)**	**Section II (model 709)**	**Section II (model 709)**	**Section II (model 709)**	**Section II (model 709)**	**Section II (model 709)**
35—60	−2.4	15	11	3	35—60	−2.4	15	11	3.0

Depth, km	dv/dz, 10^{-3} sec^{-1}	dT/dz, °C/km—$\beta_l \times 10^3$ 0.5	dT/dz, °C/km—$\beta_l \times 10^3$ 0.7	dT/dz, °C/km—$\beta_l \times 10^3$ 2.5	Depth, km	dv/dz, 10^{-3} sec^{-1}	$-dT/dz$, °C/km—$\beta_l \times 10^3$ 0.5	$-dT/dz$, °C/km—$\beta_l \times 10^3$ 0.7	$-dT/dz$, °C/km—$\beta_l \times 10^3$ 2.5
60—90	−5.3	21	15	4	60—110	0	10	7.4	2.0
90—150	3	7.8	5.6	1.6	110—150	2.5	8.8	6.3	1.8
150—250	2.7	8.4	6.0	1.7	150—220	5.4	3.0	2.1	0.6
250—300	5.2	3.4	2.4	0.7	220—300	5.1	3.6	2.6	0.7
300—380	6.25	1.4	1.0	0.3	300—390	3.2	7.4	5.3	1.5
380—500	4.3	7.8	5.6	1.6	390—500	4.7	7.0	5.0	1.4

Let us now consider the values of the temperature gradients that must be determined by the thermal parameters: heat sources, thermal conductivity, and heat flux. The set of thermal solutions is determined by the equation

$$\frac{d}{dz}\left[\lambda(z)\frac{dT}{dz}\right] = -H(z). \tag{3}$$

The source function $H(z)$ may be specified in the form of an exponentially decreasing function with depth or as a piecewise-constant one. The problem is defined if

$$\int_z^h H(z)\,dz = Q, \quad Q(h) = 0, \quad Q(z=0) = Q_0,$$

where Q_0 is the heat flow observed at the surface. The solution has the form

$$\frac{dT}{dz} = \left\{Q - \frac{H_0}{a}\left[1 - \exp(-az)\right]\right\}/\lambda.$$

As a result, the gradient of seismic velocity can be represented through the heat flow in the form

$$\frac{dv_a}{dz} = \beta_a \left\{Q - \frac{H_0}{a}\times\right.$$

$$\left.\times[1 - \exp(-az)]\right\}/\lambda + \alpha_a \frac{dP}{dz}.$$

Fig. 3. Dependence of the temperature gradient on the magnitude of heat flow Q for different λ. The zone of measured flows in the Andes region is shaded.

Figure 2: Fig. 3. Dependence of the temperature gradient on the magnitude of heat flow Q for different λ. The zone of measured flows in the Andes region is shaded.

From Fig. 3, showing the dependence of dT/dz on Q for different λ, it follows that for the most probable $H_0 \sim 2 \cdot 10^{-13}$ cal/cm$^3 \cdot$ s, in mountainous regions with heat flow $Q = (1.5\text{-}2) \cdot 10^{-6}$ cal/cm$^2 \cdot$ s, the temperature gradient must lie, for the granite layer, within the range from 10 to 35°/km, if λ is equal to $8 \cdot 10^{-3}$ or $4 \cdot 10^{-3}$ cal/cm $\cdot$ s $\cdot$ °C. In zones of reduced heat flow, where $Q = 0.8 \cdot 10^{-6}$ cal/cm$^2 \cdot$ s, $dT/dz = (9\text{-}15)$°C/km.

Fig. 3. Dependence of the temperature gradient on the magnitude of heat flow Q for different λ. The zone of measured flows in the Andes region is shaded.
$1 - \lambda = 4,\ H_0 = 2;\quad 2 - \lambda = 4,\ H_0 = 4;\quad 3 - \lambda = 6.5,\ H_0 = 2;\quad 4 - \lambda = 4,\ H_0 = 6;\quad 5 - \lambda = 8,\ H_0 = 2;\quad 6 - \lambda = 6.5,\ H_0 = 4;\quad 7 - \lambda = 8,\ H_0 = 4;\quad 8 - \lambda = 6.5,\ H_0 = 6;\quad 9 - \lambda = 8,\ H_0 = 6.$

At the base of the granite–basalt layer (30 km), the gradient is somewhat lower than in the granite layer and reaches (18-27)°/km for $Q = 2 \cdot 10^{-6}$. The gradient $dT/dz = 12\text{-}25$°/km for $Q = 1.5 \cdot 10^{-6}$ and $dT/dz = 5\text{-}12$°/km for $Q = 1 \cdot 10^{-6}$ at the same λ. In zones of reduced heat flow (where $Q = 0.8 \cdot 10^{-6}$), $dT/dz = (4\text{-}11)$°/km. With an increase in heat generation in the granite layer, the value of dT/dz decreases. From consideration of Fig. 3 it follows that, for admissible values of heat generation and thermal conductivity, the heat-flow values for the continental crust cannot be less than $0.5 \cdot 10^{-6}$ cal/cm$^2 \cdot$ s, since otherwise the temperature gradient becomes close to zero, which is implausible.

The examples presented show that, because of the scatter of the observed thermal data, the thermal problem of finding the temperature gradient in various layers is also nonunique; only joint consideration of thermal and seismic data can substantially reduce the nonuniqueness of solving inverse problems of finding deep velocity and thermal sections.

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Note: Figure translations are in progress. See original paper for figures.