MINIMIZATION OF OXIDATION

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Abstract

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MINIMIZATION OF OXIDATION

OF THE SURFACE OF A HEATED METAL

(Presented by Academician V. A. Trapeznikov on 26 IV 1968)

The following laws of oxidation in an unchanging atmosphere at constant surface temperature are known experimentally ([1,2]):

$$w = k_\ell t + c, \tag{1}$$

$$w^2 = k_\mathrm{p} t + c, \tag{2}$$

$$w^3 = k_\mathrm{k} t + c, \tag{3}$$

$$f^w = k_\mathrm{e} t + c, \tag{4}$$

where w is the weight gain of oxidized (decarburized) metal, t is time, and $k_\ell, k_\mathrm{p}, k_\mathrm{k}, k_\mathrm{e}, c, f$ are constants.

All these laws are solutions of the differential equation

$$dw/dt = K \left| \sum_{i=0}^{n} b_i w^i \right|, \tag{5}$$

where the constant K depends on the temperature of the surface and the atmosphere.

Law (1) is obtained for $n = 0$, $b_0 = K/k_\ell$; law (2) for $n = 1$, $b_0 = 0$, $b_1 = 2K/k_\mathrm{p}$; law (3)–for $n = 2$, $b_0 = b_1 = 0$; $b_2 = 3K/k_\mathrm{k}$; law (4) is obtained for $n = \infty$, when

$$b_i = (\ln f)^{i+1}/i! \tag{6}$$

In the book ([3]) a solution is given to the problem of minimizing oxidation by choosing the graph of the surface temperature $u(t)$, $0 \le t \le T$.

We shall show that this control action $u(t)$ is universal. It ensures minimization of oxidation under any law; and also, which is practically especially important, in all cases when, in the course of heating the metal, one law is replaced by another depending on the thickness of the oxidized layer (for example, as a result of accumulation and then shedding of scale).

In the book ([3]) it is indicated that the error in minimizing oxidation, which is a consequence of the simplified mathematical description of heating of the body in terms of the surface temperature u and the mass-average temperature x of the heated product, is practically negligible. Therefore here we shall also use this description:

$$\frac{dx}{dt} = \frac{1}{\theta}(u - x), \qquad x(0) = x_0, \qquad x(T) = x_T. \tag{7}$$

Here θ is a constant determined experimentally.

We shall give the oxidation (decarburization) law the general form

$$dw/dt = \psi(u)/\varphi(w); \qquad w(0) = w_0, \tag{8}$$

where ψ, as follows from the properties of oxidation processes ([1,2]), is a known

a continuous positive definite monotone function. It is also essential that $\varphi(w)$ be differentiable everywhere and not equal to zero.

The problem is posed as follows: it is required to ensure satisfaction of the boundary conditions (7), obtaining in this case

$$\min_{u(t),\, 0 \le t \le T} w(T). \tag{9}$$

We shall use the maximum principle [4]:

$$H = \frac{p_1}{\theta}(u - x) + p_2 \frac{\psi(u)}{\varphi(w)}, \tag{10}$$

where

$$\dot{p}_1 = -\frac{\partial H}{\partial x} = \frac{p_1}{\theta}; \qquad p_1 = \mu e^{t/\theta} \tag{11}$$

(μ is an unknown constant);

$$\dot{p}_2 = -\frac{\partial H}{\partial w} = p_2 \psi(u) \frac{d\varphi(w)/dw}{\varphi^2(w)} = p_2 \frac{dw}{dt} \frac{d\varphi(w)/dw}{\varphi(w)}. \tag{12}$$

From (12) and (7) it follows that

$$p_2 = -c_1 \varphi(w), \tag{13}$$

where $c_1 > 0$ is a constant.

Substituting (11) and (13) into (10), we obtain:

$$H = \frac{\mu}{\theta} e^{t/\theta}(u - x) - c_1 \psi(u). \tag{14}$$

From the condition that H be maximal with respect to u, it follows that

$$\frac{d\psi(u)}{\partial u} \equiv \Phi(u) = \frac{\mu}{c_1 \theta} e^{t/\theta} = c_2 e^{t/\theta}. \tag{15}$$

Thus,

$$u = \Phi^{-1}(c_2 e^{t/\theta}), \tag{16}$$

where the constant c_2 is found from the solution of the boundary-value problem (7) and, as is already evident from (14), does not depend on the function $\varphi(w)$, i.e., has one and the same value for any oxidation law of type (5).

Thus, the minimization algorithm for oxidation presented in the book [3] is suitable for all laws that can be reduced to the form (8), in particular, for laws (1)–(4). This means that it is possible to create a programmed-control system implementing the control (16). The settings of the programming device should be the quantities θ, x_0, x_T, and T. Such a system guarantees minimization of oxidation if the function $\psi(u)$ and the parameter θ have first been obtained experimentally.

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Note: Figure translations are in progress. See original paper for figures.