ON THE PERTURBATION THEORY FOR POLARIZATION IN RESONANT MEDIA

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ON THE PERTURBATION THEORY FOR POLARIZATION IN RESONANT MEDIA

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One of the principal problems in the theory of optical quantum generators (OQG) consists in calculating the nonlinear polarization of matter. For these purposes one usually uses the equations of motion for the one-particle density matrix ρ, describing the system of radiating atoms. Into these equations there enters, as a perturbation, the classical electromagnetic field inside the resonator under consideration. Higher approximations of perturbation theory for the density matrix make it possible, by known formulas ([1]), to calculate the macroscopic polarization and thereby to find the nonlinear terms in the equations of oscillation of the electromagnetic field in the resonator. It is assumed here that the criterion for applicability of the theory is the smallness of the parameter $\varepsilon = \eta - 1$, where $\eta = N/N_{\text{thr}}$ is the relative pumping. In the literature one usually confines oneself to the third approximation of perturbation theory for the off-diagonal elements of the density matrix. The question of convergence of the perturbation-theory series has not yet been investigated in sufficient detail.

Finding higher approximations for the density matrix by the usual method of successive approximations ([1]) appears to be very cumbersome. In this connection it is expedient to use the diagram technique for the density matrix ([2]), the basic principles of which, as applied to the problem under consideration, are formulated below.

Let us consider the density matrix

$$\rho_{m\mu}(t) = \langle a_\mu^+(t) a_m(t) \rangle, \tag{1}$$

where a_m and a_μ^+ are Heisenberg annihilation and creation operators of a particle at the upper and lower laser levels; the indices m and μ refer to the corresponding Zeeman sublevels. The averaging in expression (1) is performed over some initial distribution. The operator of the interaction energy of the atomic system with the electromagnetic field can be written in the form

$$H_1(t) = -\sum_{m\mu}(d_{m\mu}E(r,t))a_\mu^+ a_m \exp(-i\omega_{m\mu}t) + \text{c.c.}, \tag{2}$$

where $d_{m\mu}$ denotes the matrix elements of the dipole moment between the lower and upper states, and $E(r,t)$ is the electric-field strength.

Numerical calculations of the terms of the fifth approximation of perturbation theory were carried out as applied to the linear Zeeman effect for a gas laser placed in an axial magnetic field. In this case the electromagnetic field has the form of standing waves with right and left circular polarizations, and the electric field in the single-mode approximation can be described by the quantities $E^{(+)}$ and $E^{(-)}$:

$$E^{(+)} = E^{(-)*} = \frac{1}{\sqrt{2}}(E_x + iE_y) = (E_1 e^{i\nu_1 t} + E_2 e^{-i\nu_2 t})\sin K_n z, \tag{3}$$

where E_1 and E_2 are amplitudes; ν_1 and ν_2 are the frequencies of waves with right- and left-circular polarizations; $\Omega_n = cK_n$ is the eigenfrequency of the empty resonator.

The subsequent transformations of expression (1) are connected with the transition from the Heisenberg representation to the interaction representation, with the following transformation of the product of operators into the form of a normal product. In the course of these transformations, contractions of the field operators will correspond to various internal electron lines, which will be located on two sections of the contour. The latter will henceforth be called the upper and the lower sections. In the n-th approximation of perturbation theory the diagrams will contain n external photon lines ending at points with time arguments $t_1, t_2, \ldots, t_n$. If the number of external lines on the upper contour is m, and on the lower one $n-m$, then it will be necessary to take into account all $n!/m!(n-m)!$ diagrams corresponding to different mutual arrangements of the points on the upper and lower sections of the contour. The contribution of any one of the diagrams can be found on the basis of the following rules:

a) the product of all exponential factors $\exp(-i\omega_{mk}t)$ entering into the product of the operators $H_I(t_1)\ldots H_I(t_n)$ can be written in the form

$$\exp\left(-i\sum_{k=1}^{n}\omega_{st}\tau_k\right),$$

where τ_k is the time interval between two neighboring points, $\tau_k = t_{k-1} - t_k$, and ω_{st} is the frequency difference in the vertical section over the interval $[t_{k-1}, t_k]$, with $\hbar\omega_s$ the energy assigned to the upper lines and $\hbar\omega_t$ the energy assigned to the lower lines;

b) allowance for proper energy contributions on any of the segments of the upper or lower sections of the contour leads to the appearance of factors of the form $\exp\{-\gamma(t_k - t_l)\}$. If the product of these factors is transformed to a form containing only differences of the intervals τ_k, then in each vertical section the constants γ will be added;

c) if only the most significant resonance terms are taken into account, then each of the frequencies $\omega_{st} > 0$ will enter the formulas in combination with one of the frequencies ν_i in the form $\omega_{st} - \nu_i$. The frequency ν_i must be chosen, bearing in mind the case of a three-level system that will interest us below, as follows (3). If in a vertical section the indices s and t belong to different levels, then for $\Delta m_{st} = +1$ the frequency $\nu_i = \nu_1$, while for $\Delta m_{st} = -1$ it will be equal to $\nu_i = \nu_2$. If s and t denote Zeeman sublevels of one level, then for $\Delta m_{st} > 0$, ν_i is equal to $\nu_i = \Delta\nu = \nu_1 - \nu_2$, and $\nu_i = -\Delta\nu$ for $\Delta m_{st} < 0$.

The calculation of the macroscopic polarization of the medium makes it possible to find equations for the amplitudes E_1 and E_2

$$\dot{E}_1 = E_1\{1/2\,\gamma/Q - \chi_1(E_1^2, E_2^2)\}, \qquad \dot{E}_2 = E_2\{1/2\,\gamma/Q - \chi_2(E_1^2, E_2^2)\}. \quad (4)$$

The expansion of χ_1 and χ_2 in powers of E_1^2 and E_2^2 will have an alternating-sign character,* i.e., the terms of the third approximation in (4) will enter with a minus sign, the terms of the fifth approximation with a plus sign, etc. In the fifth approximation of perturbation theory, equations (4) have the following form

$$\begin{aligned} \dot{E}_1 &= a_1 E_1 - \beta_1 E_1^3 - \theta_{12} E_1 E_2^2 + \gamma_1 E_1^5 + \Delta_{12} E_1 E_2^4 + \lambda_{12} E_1^3 E_2^2, \\ \dot{E}_2 &= a_2 E_2 - \beta_2 E_2^3 - \theta_{21} E_2 E_1^2 + \gamma_2 E_2^5 + \Delta_{21} E_2 E_1^4 + \lambda_{21} E_2^3 E_1^2, \end{aligned} \quad (5)$$

where the expansion coefficients are functions of the strength of the magnetic field H. The calculation of these coefficients was performed for the case in which the quantum numbers of the excited and ground states are equal—

* Under the assumption of not too large amplitudes of the magnetic field and detunings δ between the resonator frequency and the center of the Doppler line.

were $j_m = 1$, $j_\mu = 0$. For each of the elements of the density matrices $\rho_{1,0}$ and $\rho_{-1,0}$, 16 diagrams of the fifth approximation were taken. All integrals that arise can, in the general case, be expressed in terms of the special functions $w(z)$, whose basic properties and detailed tables are given in (4). In the numerical calculations, in accordance with the adopted values $\gamma_m/K_{nU} = 0.1067$, $\gamma_\mu/K_n/U = 0.0133$, the Doppler approximation $\gamma/K_{nU} \ll 1$ was used, where U is the root-mean-square velocity of the Maxwellian distribution.

Fig. 1
Fig. 1

Fig. 2
Fig. 2

In Figs. 1 and 2, for the case $\delta = 0$, the computed stable stationary values of the dimensionless wave intensity I and the beat frequency $\Delta\nu$ (in units of $\nu/2Q$) are given for various pump values as functions of the parameter

$$x = \frac{\omega_{1,0} - \Omega_n}{K_{nU}},$$

which is proportional to the magnetic-field strength. The dashed line shows the corresponding quantities, borrowed from previous calculations (5), in the third order of perturbation theory. The sign-changing character of the dependence of $\Delta\nu$ on H in experiments on the Zeeman effect in gas lasers is well known (6). We note that, when terms of fifth order in perturbation theory are taken into account, the curves of the dependence of $\Delta\nu$ on H for different η no longer intersect at one point (5), but instead there is a certain region of intersection points. In this region, at fixed H, the beat frequency $\Delta\nu$ depends nonmonotonically on η.

On the basis of the calculations performed, one may conclude that for relatively small pump values $\varepsilon < 0.10$ the contribution of the fifth-approximation terms is small, and therefore the usually used third approximation is quite justified. As the pump is increased, beginning with a certain critical value ε_{cr}, the system of equations (5) no longer has stationary real solutions. For the chosen parameters $\varepsilon_{\text{cr}} = 0.20$. This conclusion is also confirmed by the calculations of Uehara and Shimoda (7) for a two-level system. In the seventh approximation, real solutions arise also for $\varepsilon \geq 0.20$. All this indicates that the convergence of the perturbation-theory series at pump values $\eta \geq 1.15$ becomes poor in Lamb' s theory, i.e., the upper limits of applicability of the usually used theory (1) are bounded by pump values of the order $\eta \simeq 1.15$. This last number, of course, depends somewhat on the parameters of the problem, in particular on the values of γ_m and γ_μ; nevertheless, when the latter are varied within reasonable limits, the approximate values of the critical pump values are preserved.

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Note: Figure translations are in progress. See original paper for figures.