ON THE PROBLEM OF DETERMINING CONTROL FORCES IN NONLINEAR SYSTEMS

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Abstract

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ON THE PROBLEM OF DETERMINING CONTROL FORCES IN NONLINEAR SYSTEMS

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Ya. N. Roitenberg solved the problem of determining control forces that transfer controlled systems from one position in phase space to another over a prescribed time interval ([1]).

If the equation of motion of the system has the form

$$\dot{u} = Au + Q_0(t),$$

where $Q_0(t)$ is the control vector to be determined, then the general solution of this system can be written in the form

$$u(t) = N(t)u(0) + \int_0^t N(t-\tau)Q_0(\tau)\,d\tau. \tag{1}$$

Here $N(t)$ is the fundamental matrix of the corresponding homogeneous system, and $N(t-\tau) = N(t)N^{-1}(\tau)$ is its weight function.

According to (1), in order to determine the control vector the prescribed time interval $[0, T]$ is divided into a certain number of subintervals, on each of which the values of the components of the vector $Q_0(t)$ are assumed constant. The number of subintervals must be such that the total number of steps of all nonzero components of the vector $Q_0(t)$ is equal to the order of the system. This condition makes it possible to determine the control vector from the given class of functions, for the given method of partitioning the prescribed time interval, in a unique way. The column matrix q_{0l}, whose components are the constant values of the sought vector $Q_0(t)$ on each of the subintervals of the partition of $[0, T]$, is determined by the equality

$$q_{0l} = -BN(T)u(0),$$

where B is the matrix inverse to the matrix of the algebraic system of equations obtained from (1) under the condition $u(T) = 0$.

Below, one of the possible iterative processes is considered for determining the above-mentioned control vector for nonlinear systems.

Let there be given the vector differential equation

$$\dot{z} = \varphi(z) + Q(t), \tag{2}$$

where $\varphi(z)$ is a certain continuous nonlinear vector-function, which is defined in a closed region R containing the point $z(0)$; $Q(t)$ is a control vector depending only on t, whose components are step functions. The initial value $z(0) \in R$.

It is required to bring system (2), at the time $t = T$, to the origin of coordinates $z(T) = 0$.

As the norm of the vector $z(t)$ we take

$$\|z(t)\| = \sup_{i=1,2,\ldots,r} |z_i|,$$

and as the norm of the matrices $P = (p_{ij})$, the quantity

$$\|P\| = \sup \sum_{j=1}^{r} |p_{ij}|.$$

Theorem. If the following conditions are satisfied:

a) the vector function $\varphi(z)$ in the domain R, where $\|z\| \leq A_0$, is representable in the form

$$\varphi(z) = Az + f(z) \qquad (f(0) = 0);$$

b) the nonlinear vector function $f(z)$ in the domain R satisfies the Lipschitz conditions in the norm

$$\|f(z_1) - f(z_2)\| \leq m\|z_1 - z_2\|$$

with a constant m common to the whole domain;

c) $\|N(t)\| \leq C,\ \|B\| = \alpha$;

d) the quantities m, α, C, T satisfy the inequality

$$\chi\mu e^{\mu} < 1 \qquad (\chi = C\alpha T,\ \mu = CmT);$$

e) the norm of the initial conditions $\|z(0)\|$ satisfies the condition

$$\|z(0)\| < A_0/A_1,$$

where

$$A_1 = \left\{C + \chi\left[C + \frac{C\mu e^{\mu}(1+\chi)}{1-\chi\mu e^{\mu}}\right]\right\}e^{\mu},$$

then, for the nonlinear system (2), a convergent iterative process can be constructed which determines, in the limit, the control vector $Q^*(t)$, generating the unique solution $z^*(t)$ satisfying the prescribed boundary conditions $z^*(0) = z(0),\ z^*(T) = 0$.

Indeed, the conditions of the theorem make it possible to construct such a convergent functional sequence $\{Q_n(t)\} \to Q^*(t)$, which determines the sequence of functions $\{z_n(t)\} \to z^*(t),\ z_n(t) \in R,\ z^*(t) \in R,\ t \in [0, T]$, satisfying the differential equation

$$\dot{z}^*(t) = \varphi[z^*(t)] + Q^*(t)$$

and the boundary conditions $z^*(0) = z(0),\ z^*(T) = 0$.

The iterative process proposed for proving the theorem is described by the recurrent relations

$$N(T)z(0) + \int_0^T N(T-\tau)f[z_n(\tau)]\,d\tau + \int_0^T N(T-\tau)Q_n(\tau)\,d\tau = 0, \qquad (3)$$

$$z_{n+1}(t) = N(t)z(0) + \int_0^t N(t-\tau)f[z_{n+1}(\tau)]\,d\tau + \int_0^t N(t-\tau)Q_n(\tau)\,d\tau.$$

The first relation (3) determines the control vector $Q_n(t)$ at the n-th step, while the second determines the $(n+1)$-st approximation to the solution of the nonlinear system (2). From (3) it follows that $Q_n(t) = Q_{n-1}(t) + q_n(t)$, where $q_n(t)$ is determined by the formula

$$q_n = -Bz_n(T).$$

The following estimates are valid:

$$\|z_n(t)\| \le A_1^{(n)}\|z(0)\|,$$

$$\|z_n(T)\| \le A_2^{(n)}\|z(0)\|,$$

$$\|q_n(t)\| \le \alpha A_2^{(n)} \|z(0)\|,$$

$$\|Q_n(t)\| \le \alpha A_3^{(n)} \|z(0)\|,$$

$$\|z_n(t) - z_{n-1}(t)\| \le A_4^{(n)} \|z(0)\|.$$

$(n = 1, 2, ...)$, where

$$\begin{aligned} A_1^{(n)} &= (C + \varkappa A_3^{(n-1)}) e^{\mu} && (n = 1, 2, ...), \\ A_2^{(n)} &= \varkappa \mu e^{\mu} A_2^{(n-1)} && (n = 2, 3, ...), \\ A_3^{(n)} &= A_3^{(n-1)} + A_2^{(n)} && (n = 1, 2, ...), \\ A_4^{(n)} &= \varkappa e^{\mu} A_2^{(n)} && (n = 2, 3, ...), \end{aligned}$$

in particular,

$$A_3^{(0)} = C, \qquad A_2^{(1)} = C\mu e^{\mu}(1 + \varkappa), \qquad A_4^{(1)} = A_1^{(1)}.$$

Condition c) of the theorem is sufficient for the convergence of the given iterative process, whence:

$$\lim_{n\to\infty} A_1^{(n)} = \left\{ C + \varkappa \left[C + \frac{C\mu e^{\mu}(1+\varkappa)}{1 - \varkappa \mu e^{\mu}} \right] \right\} e^{\mu} = A_1$$

$$\lim_{n\to\infty} A_2^{(n)} = 0,$$

$$\lim_{n\to\infty} A_3^{(n)} = \left(C + \frac{C\mu e^{\mu}(1+\varkappa)}{1 - \varkappa \mu e^{\mu}} \right),$$

$$\lim_{n\to\infty} A_4^{(n)} = 0.$$

It follows from this that

$$\lim_{n\to\infty} z_n(t) = z^*(t), \qquad \lim_{n\to\infty} z_n(T) = 0, \qquad \lim_{n\to\infty} Q_n(t) = Q^*(t),$$

$$\lim_{n\to\infty} q_n(t) = 0, \qquad \lim_{n\to\infty} [z_n(t) - z_{n-1}(t)] = 0.$$

It is necessary to note that condition d) ensures the convergence of the iterative process, since condition c) of the theorem will certainly be satisfied.

The iterative process described was tested on a number of applied problems.

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REFERENCES

1. Ya. N. Roitenberg, *Some Problems of Motion Control*, Moscow, 1963.

Note: Figure translations are in progress. See original paper for figures.