THREE-BEAM INTERFERENCE OF ELECTRONS BY MEANS OF AN ELECTROSTATIC PRISM

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THREE-BEAM INTERFERENCE OF ELECTRONS BY MEANS OF AN ELECTROSTATIC PRISM

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To realize three-beam interference of electrons in the UEMV-100 electron microscope, operating in the mode of an interference instrument ([1]), an electrostatic prism with two filaments is installed. The prism is a grounded capacitor, between the plates of which two parallel metallized quartz filaments of diameter $\sim 1\mu$ are placed. The filaments lie in a plane that has a small inclination α to the optical axis. The prism is placed between the objective and intermediate lenses in the specimen-holder stage. A two-lens condenser and the microscope objective are used as the illuminator. A slit diaphragm with a slit width of 5–10μ is placed in the long-focus condenser. The imaging system is a projection lens that magnifies the interference pattern by a factor of 250–300. The distances from the prism to the plane of the sources and to the observation plane are, respectively, $l_1 = 120$ mm and $l_2 = 180$ mm.

An electrostatic prism with two filaments splits the front of an electron wave coming from a coherent source into three wave fronts which, when superposed in the observation plane, give a pattern of three-beam interference*. The law of variation of intensity in the three-beam electron interference pattern, for equal distances between the virtual sources, is given by the expression

$$A^2 = 3B_0 + 4B_1 \cos\psi + 2B_2 \cos 2\psi. \tag{1}$$

Here $\psi = \dfrac{2\pi}{\lambda}\beta d$, where λ is the electron wavelength, β is the angle between the direction of wave propagation and the optical axis of the instrument, and d is the distance between the virtual sources.

$$B_0 = \int_{-\infty}^{+\infty} f(x \ )\,dx \ ,$$

where $f(x \ )$ is the intensity of an elementary wave issuing from the source point x ; B_0 is the background intensity outside the interference zone;

Fig. 1

Figure 1: Fig. 1

Fig. 3

Figure 2: Fig. 3

$$B_1 = \int_{-\infty}^{+\infty} f(x \quad) \cos \left(\frac{2\pi}{\Delta x} \frac{l_2}{l_1} x \quad \right) dx \quad ,$$

where $\Delta x = \lambda(l_1 + l_2)/d$ is the spacing between maxima in the three-beam interference pattern, and

$$B_2 = \int_{-\infty}^{+\infty} f(x \quad) \cos 2 \left(\frac{2\pi}{\Delta x} \frac{l_2}{l_1} x \quad \right) dx \quad .$$

* The light-optical analog of such a prism is a glass prism whose transverse section has the form of a trapezoid.

Fig. 1. Series of interference patterns produced by an electrostatic prism with two filaments at different prism inclination angles α. $U = 75$ kV, $U_{\text{n}} = 7$ V. Electron-optical magnification, 260; optical magnification, 20. a—separate image of two-beam interference patterns; $\alpha = 6 \cdot 10^{-3}$ rad.; , , —different degrees of superposition of two-beam patterns, respectively at angles $3.3 \cdot 10^{-3}$, $2.5 \cdot 10^{-3}$, and $1.3 \cdot 10^{-3}$ rad.; —three-beam interference pattern, $\alpha = 6 \cdot 10^{-4}$ rad.; , , —formation of a two-beam interference pattern with doubled fringe frequency upon decrease of the angle; —pattern of two-beam interference with doubled fringe frequency, $\alpha = 0$.

Fig. 3. Pattern of three-beam electron interference in the presence of beats and intensity modulation along the fringes.

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Equation (1) does not take into account the modulation of the intensity of the interference pattern caused by Fresnel diffraction at the threads. However, this does not change the fundamental essence of the phenomena under consideration.

For the case of a point source, equation (1) takes the form

$$A^2 = B_0(3 + 4\cos\psi + 2\cos 2\psi). \quad (2)$$

The maximum value of the intensity is equal to $9B_0$ instead of $4B_0$ for two-beam interference [2]. Between two maxima located at the same distance from one another as in two-beam interference, there is an additional maximum of magnitude B_0.

Figure 1 shows a series of interference patterns obtained with an electrostatic prism with two threads at different angles of inclination of the prism α. At a large angle α, the prism with two threads is, as it were, a system of two Fresnel biprisms. Therefore, two two-beam interference patterns arise in the observation plane (Fig. 1a). As the angle α is decreased, the interference patterns shift toward the optical axis and overlap—there is a transition from two-beam (Fig. 1a) to three-beam (Fig. 1d) interference. Three-beam interference in pure form (Fig. 1d) occurs when the angular width of the beam passing between the threads is equal to the angular width of the zone of two-beam interference. The maxima of three-beam interference are narrower, and their intensity is 1.4 times greater than in the case of two-beam interference. In the ideal case, the intensity of three-beam interference should be greater by a factor of 2.25. Apparently, this discrepancy is due to the finite dimensions of the illumination source. The periodicity of the fringes in three-beam interference is the same as in two-beam interference.

It is seen from Fig. 1 that at angles $\alpha < 6 \cdot 10^{-4}$ rad (the angle corresponding to the maximum zone of three-beam interference), regions with doubled fringe frequency appear in the interference patterns and gradually spread over the entire interference field. This is explained by the fact that, as the angle α is decreased, the width of the beam passing between the threads decreases, which first leads to a decrease of the zone of three-beam interference, and then to its complete disappearance. In this case the interference pattern is a pattern of two-beam interference from two imaginary sources located from one another at the doubled distance $2d$. As a result, the spacing between the fringes of the interference pattern is half as large.

To estimate the quality of the interference pattern, the concept of visibility was used, introduced into light optics by Michelson:

$$K = (A^2_{\max} - A^2_{\min})/(A^2_{\max} + A^2_{\min}). \tag{3}$$

Starting from (1), we obtain

$$K = 8B_1/(6B_0 + 4B_2). \tag{4}$$

For the particular case in which the source is a uniformly illuminated slit of width δ, we have

$$K = \frac{8\sin\xi/\xi}{6 + 4\sin\xi/2\xi}. \tag{5}$$

Here

Fig. 2. Visibility functions for three-beam and two-beam electron interference.

Figure 3: Fig. 2. Visibility functions for three-beam and two-beam electron interference.

$$\xi = 2\pi \frac{\delta}{\Delta_x} \frac{l_2}{l_1}.$$

The character of the variation of the visibility function K with ξ is shown in Fig. 2.* Also given there is the visibility function calculated by us for two-beam interference,

$$K = \sin\xi/\xi.$$

Experimental visibility functions—

* The visibility function for three-beam interference at $\xi = 0$ is equal to 0.8. This is because the additional maximum between the main maxima is taken as A^2_{min}. Taking the true minima into account, the function K begins from 1 (see the dashed curve in Fig. 2).

···for two-beam and three-beam interference were constructed by bringing the minima of the experimental and theoretical visibility functions into coincidence along the abscissa axis. Since Fresnel diffraction at the filaments modulates the interference pattern, the average value of the visibility measured for many fringes was taken as the experimental value.

Fig. 2. Visibility functions for three-beam and two-beam electron interference. **1**—theoretical curve for three-beam interference; **2**—experimental curve for three-beam interference; **3**—theoretical curve for two-beam interference; **4**—experimental curve for two-beam interference.

The presence of secondary maxima in the experimental visibility functions indicates that the brightness distribution in the source differs from a Gaussian distribution, which does not give secondary maxima in the visibility function. However, the experimental curves differ noticeably from the theoretical ones calculated also for the case of a source with a uniform brightness distribution. Apparently, the real source is an image of the target with blurred edges. It is not possible to obtain the brightness distribution in the source from the experimental visibility functions without a preliminary choice of a source model. One can only estimate the effective size of the source. In our experiment it proved to be equal to 1700 Å.

If the sources S_1, S_2, and S_3 are at different distances from one another and do not lie on one straight line, then beats and intensity modulations along the fringes arise in the interference pattern. Let us consider the law of intensity variation in a three-beam interference pattern for this case. Let the imaginary point sources S_1 and S_3 lie on the x axis at distances d_1 and d_3 from the origin, and let S_2 lie on the y axis at a distance d_2 from the origin, with $d_2 \ll (d_1 + d_3)$. Then the expression for the intensity of the three-beam interference pattern has the form

$$A^2 = B_0 \left[3 + 4\cos\left(\frac{2\psi_2 + k\psi_1}{2}\right) \cos(2+k)\frac{\psi_1}{2} + 2\cos(2+k)\psi_1 \right]. \qquad (6)$$

Here $\psi_1 = \dfrac{2\pi}{\lambda} d_1 \dfrac{X}{l}$, $\psi_2 = \dfrac{2\pi}{\lambda} d_2 \dfrac{Y}{l}$, where X and Y are the coordinates of the point under consideration in the observation plane, located at a distance l from the sources (the Y axis coincides with the direction of the fringes), and $k = (1 - d_3/d_1)$. Equation (6) takes into account both beats and modulation along the fringes. The position of the maxima in the interference fringes is found from the condition $\partial A^2/\partial \psi_2 = 0$. After simple transformations we obtain

$$Y = \frac{d_3 - d_1}{2d_2} X - \frac{n\lambda l}{d_2}. \qquad (7)$$

This equation represents a system of straight lines in the observation plane of the interference, located along the Y axis at distances $\Delta_\perp = \lambda l/d_2$ from one another. The inclination of these straight lines to the Y axis, i.e., to the direction of the interference–

of the bands is equal to $\theta = \operatorname{arcctg}(d_3 - d_1)/2d_2$. For $d_3 = d_1$, $\theta = \pi/2$, i.e., we have the case of pure modulation along the bands. For $d_2 = 0$, $\theta = 0$, we have the case of pure beats. Taking $d = (d_1 + d_3)/2$, one can easily obtain expressions for the angle between the filaments

$$\varphi = 2\frac{\Delta_x}{\Delta_\perp} \qquad (8)$$

and for the relative difference in the distances between the sources

$$\frac{\Delta d}{d} = 2\frac{\Delta_x}{\Delta_\perp} \operatorname{ctg}\theta. \qquad (9)$$

Figure 3 shows a pattern of three-beam interference arising in the presence of beats and modulation along the bands. The regions in which maxima are transformed into minima are clearly visible. These regions have an inclination θ to the direction of the bands, and the distance between the regions is equal to $^1/_2\Delta_\perp$. The presence of modulation along the bands indicates that the source also has small dimensions in the direction of the bands. By measuring Δ_x, $\Delta_\perp$, and θ on the interference pattern, one can determine the angle between the filaments φ and the relative difference in the distances between the virtual sources $\Delta d/d$.

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Note: Figure translations are in progress. See original paper for figures.