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Flow of a Viscous Gas Near a Wall with a Break

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1. In the present work the parameters are sought of a supersonic flow of a viscous compressible gas near a wall with a break (Fig. 1). To solve the problem, the entire region Q of the flow, bounded by the wall of the angle ABC and by the half-line $AD \perp AB$, is divided into the subregions Q_1, Q_2, and Q_3.

In the neighborhood Q_1 of the angular point no assumptions are made about the orders of the unknown functions and their gradients; therefore, in Q_1 the full system of Navier–Stokes equations for a compressible viscous gas is solved.

The region Q_2 is located sufficiently far from the wall that the flow in Q_2 satisfies, to an accuracy of $o(\varepsilon^2) = o(1/\mathrm{Re})$, the equations of an inviscid gas (1)–(4)

$$\operatorname{rot} \mathbf{V} = 0; \tag{1}$$

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0; \tag{2}$$

$$u\frac{\partial}{\partial x}\left(\frac{P}{\rho^\gamma}\right) + v\frac{\partial}{\partial y}\left(\frac{P}{\rho^\gamma}\right) = 0; \tag{3}$$

$$u\frac{\partial}{\partial x}\left(\frac{u^2+v^2}{2}\right) + v\frac{\partial}{\partial y}\left(\frac{u^2+v^2}{2}\right) + \frac{1}{\rho}\left(u\frac{\partial P}{\partial x} + v\frac{\partial P}{\partial y}\right). \tag{4}$$

Fig. 1. Flow near a wall with a break

$$Q = Q_1 + Q_2 + Q_3$$

In the present work it is immaterial that in Q_2 stationary equations are used, while in Q_1 and Q_3 nonstationary ones are used, since we shall be interested only in the limit of the solution as $t \to \infty$.

Q_3 is the region near the wall, but sufficiently far from the angular point that the longitudinal gradients of the unknown functions in Q_3 are quantities of order $o(1)$; i.e., the flow satisfies, to an accuracy of $o(\varepsilon^2)$, equations (5)–(8).

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{\partial P}{\partial x}+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right); \tag{5}$$

$$\rho\frac{\partial v}{\partial t}+\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}=-\frac{\partial P}{\partial y}+\frac{4}{3}\frac{1}{\mathrm{Re}}\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{2}{3}\frac{1}{\mathrm{Re}}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial x}\right); \tag{6}$$

$$\rho\frac{\partial T}{\partial t}+\rho u\frac{\partial T}{\partial x}+\rho v\frac{\partial T}{\partial y}=-(\gamma-1)\gamma M_0^2P\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+$$

$$+\frac{\gamma}{\mathrm{Re}\,\mathrm{Pr}}\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+\frac{M_0^2\gamma(\gamma-1)}{\mathrm{Re}}\mu\left(\frac{\partial u}{\partial y}\right)^2; \tag{7}$$

$$\frac{\partial\rho}{\partial t}+\frac{\partial\rho u}{\partial x}+\frac{\partial\rho v}{\partial y}=0; \tag{8}$$

u, v are the longitudinal and transverse components of velocity; P, ρ, T, μ, λ are the pressure, density, temperature, and the coefficients of viscosity and thermal conductivity; γ, Re, Pr, M_0 are the ratio of specific heats and the Reynolds, Prandtl, and Mach numbers. In forming the dimensionless quantities, the parameters of the oncoming flow are used; as the characteristic length l_0 is taken

thickness of the oncoming boundary layer AD; x and y are the longitudinal and transverse coordinates, chosen so that the boundary Γ_2 is the line $x = C$.

The position of the boundaries Γ_1 and Γ_2 is not known in advance; however, the accuracy of the solution in Q is not worsened when Q is enlarged at the expense of Q_2 and Q_3, since in Q_1 the full system is solved. We take Q_1 to be deliberately larger than required by the available a priori estimates.

The results of the numerical calculation confirm the correctness of the choice of the boundaries Γ_1 and Γ_2 made in the paper. For simplicity of the calculations it is expedient to choose Γ_1 so that the flow above Γ_1 is vortex-free. Therefore the shape of Γ_1 was chosen so that the streamlines at the points of Γ_1 are directed into Q_1, if the direction of the oncoming flow from left to right is taken as positive. In this connection the shape of $\Gamma_1(g(x))$ was determined as the solution of the equation

$$dg/dx = v^*(x,g)/u^*(x,g)+\chi;$$

u^*, v^* are the longitudinal and transverse components of velocity at the points (x, g), corresponding to inviscid flow past the same angle; χ is a certain positive constant.

2. To solve the Navier–Stokes equations in the cylinder $Q_1 \times t$, it is necessary to specify initial and boundary conditions. We were interested only in the steady solution, i.e., its limit as $t \to \infty$; therefore the specification of the initial conditions is to a considerable extent arbitrary.

The conditions on Γ_4 are the known parameters of the oncoming flow, which constitute the solution of the boundary-layer equations for a compressible gas on a plate (the transverse velocity component is set equal to zero) ([1]); on Γ_3 there are the usual no-slip conditions and a known wall temperature; the density at the wall was found from the equation of continuity ([2]). The boundary conditions on Γ_1 were found from the conditions of smooth matching of the solutions of the Navier–Stokes system in Q_1 and of the inviscid-gas equations in Q_2 ([3]). In accordance with the form of Γ_1, it was required to prescribe on it 5 conditions of smooth matching. The first 4 are the continuity on Γ_1 of the sought functions, i.e., the velocity components, temperature, and density. As the fifth, the condition of continuity on Γ_1 of the left-hand side of equation (1) is taken, i.e., the condition of continuity of the velocity vorticity.

The choice of equation (1) from the system (1)–(4) is uniquely determined by the fact that the additional condition of continuity, for any possible values of the sought functions in the given problem, must not be a consequence either of the first 4 continuity conditions or of the equations describing the flow in Q_1 and Q_2.

The flow in Q_2 is a simple wave, i.e., the quantities u, v, ρ, T are functions of one parameter, for example, of the longitudinal velocity component u; $v = f_1(u)$; $\rho = f_2(u)$; $T = f_3(u)$ ([4]). But the flow parameters are continuous on Γ_1; consequently, in $\bar{Q}_1$ on Γ_1 the same relations are valid. In Q_2, $\operatorname{rot} V = 0$; consequently, also in Q_1 on Γ_1, $\partial u/\partial y - \partial v/\partial x = 0$.

Thus, on Γ_1 in $\bar{Q}_1$ we have the following boundary conditions for the Navier–Stokes system:

$$\partial u/\partial y - f_1'(u)\partial u/\partial x = 0; \tag{9}$$

$$v = f_1(u); \tag{10}$$

$$\rho = f_2(u); \tag{11}$$

$$T = f_3(u). \tag{12}$$

Fig. 2. Streamlines of variant 4)

Figure 1: Fig. 2. Streamlines of variant 4)

Fig. 3. Variation of nondimensional pressure P/P_0 for variant 3) along streamlines at different distances from the wall: 1 –along the wall; 2 –along the line $\psi = 0.05$; 3 –along the line $\psi = 0.6$; 4 –along the line $\psi = 0.85$

Figure 2: Fig. 3. Variation of nondimensional pressure P/P_0 for variant 3) along streamlines at different distances from the wall: 1 –along the wall; 2 –along the line $\psi = 0.05$; 3 –along the line $\psi = 0.6$; 4 –along the line $\psi = 0.85$

On Γ_2, 7 conditions of smooth matching of the solutions of the Navier–Stokes equations and equations (5)–(8) are specified. These are the conditions of continuity of the functions u, v, ρ, T and of the left-hand sides of equations (5)–(7).

The choice of equations (5)–(8), which differ from the classical Navier–Stokes equations, is explained by the requirements for the solvability of the resulting

boundary-value problem in Q_1. In addition, in equations (5)–(8) all terms describing the inviscid flow are retained; this is important, since at large Mach numbers part of the boundary Γ_2 lies in the region of the expansion wave; finally, system (5)–(8) differs from the Navier–Stokes equations by terms of order $o(\varepsilon^2)$, and not $o(\varepsilon)$.

Fig. 2. Streamlines of variant 4)

If, with the above-indicated choice of the boundaries Γ_1 and Γ_2, the solution of the boundary-value problem obtained in $\overline{Q}_1$ for the system of Navier–Stokes equations exists, is unique, and depends continuously on the right-hand sides of the equations and on the boundary conditions, then it differs by a quantity $o(\varepsilon^2)$ from the exact solution of the Navier–Stokes equations in the whole domain Q. The solvability of analogous boundary-value problems for linear parabolic equations is shown in [5]. The solvability of the problem in $\overline{Q}_1$ is demonstrated by a series of numerical experiments.

3. The Navier–Stokes equations for a compressible gas with the described conditions on the boundaries of $\overline{Q}_1$ were solved numerically by the finite-difference method described in [2], on the BESM-3 computer.

Steady solutions were obtained for the following values of the Mach and Reynolds numbers and of the angle $\tilde{\alpha}$ (Fig. 1). The Reynolds number is referred to the thickness of the incident boundary layer, the Mach number to the parameters of the incident flow at infinity; the nondimensional wall temperature T_w/T_∞ is everywhere equal to 0.25.

Fig. 3. Variation of nondimensional pressure P/P_0 for variant 3) along streamlines at different distances from the wall: 1 –along the wall; 2 –along the line

$\psi = 0.05$; 3 —along the line $\psi = 0.6$; 4 —along the line $\psi = 0.85$

1) $M_0 = 1.3$; Re $= 100$; $\tilde{\alpha} = 20°$;
2) $M_0 = 1.3$; Re $= 100$; $\tilde{\alpha} = 40°$;
3) $M_0 = 2$; Re $= 100$; $\tilde{\alpha} = 20°$;
4) $M_0 = 3$; Re $= 100$; $\tilde{\alpha} = 20°$;
5) $M_0 = 4$; Re $= 400$; $\tilde{\alpha} = 20°$;
6) $M_0 = 5$; Re $= 400$; $\tilde{\alpha} = 20°$.

The shape and dimensions of the regions Q_1 are visible in the streamline plots in Fig. 2 for case 4). The dimensions and shape of the regions, as well as the streamlines for the other cases, look analogous.

The wall friction increases at the corner point; the increase is especially sharp in the case of a large angle α. The variation of pressure and temperature along the flow (see, for example, Fig. 3) shows the presence of a small compression wave ahead of an expansion wave; this wave is stronger at higher Mach numbers.

Let us consider the transverse pressure gradient $\partial P/\partial n$ along the perpendiculars to the wall after the corner point; we shall move from left to right from the corner point. On the first lines the gradient is not equal to zero; then, near the wall, a region appears where $\partial P/\partial n = 0$; it expands as one moves downstream. This agrees with the model of a sublayer growing after the corner point, proposed, for example, in ([6]). However, the vortex flow immediately above this region ($\partial P/\partial n = 0$) cannot be considered inviscid; for example, the gradients of longitudinal velocity and temperature there are large. No separation of the flow after the corner point was observed.

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Note: Figure translations are in progress. See original paper for figures.