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HEAT ENGINEERING

1967

SovietRxiv

Abstract

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UDC 536.754

HEAT ENGINEERING

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DETERMINATION OF THE MAXIMUM WORK IN AN ISOLATED SYSTEM WITH LIMITED HEAT CAPACITIES OF THE HEAT SOURCE AND THE REFRIGERATOR

(Presented by Academician Ya. B. Zel' dovich, January 23, 1967)

1. Introduction. Let us determine the limiting work capacity of a continuous sequence of thermodynamic cycles taking place between a heat source and a refrigerator of limited energy capacity. Such a sequence can be carried out in different ways. One of them occurs under nonstationary energy exchange between immobile reservoirs of heat and cold (Fig. 1), while another occurs under stationary conditions during the flow of a heat-transfer fluid and a refrigerant along a certain surface through which heat is transferred from the hot body to the cold one (Fig. 2). Part of the heat taken from the hot liquid is converted into useful work. For clarity of the discussion, we shall analyze the second case.

The problems considered are special cases of a more general problem whose aim is to determine the maximum work capacity of a thermodynamically isolated system consisting of n bodies having, at the initial instant of time, different temperatures T_i. Applying to this case the well-known Gibbs principle ([1]), according to which, in an isoentropic transition of an isolated system to a state of stable equilibrium, its energy in the final state is minimal, for the maximum work one obtains

$$L_{\max} = \sum_{i=1}^{n} m_i E_i(T_i) - \sum_{i=1}^{n} m_i E_i(T_c). \tag{1,1}$$

The quantity T_c, common to all bodies, is found from the condition

$$\sum_{i=1}^{n} m_i S_i(T_i) = \sum_{i=1}^{n} m_i S_i(T_c). \tag{1,2}$$

Here m, S, E are the mass, entropy, and energy of the i-th body.

Fig. 1. Nonstationary heat exchange.
1 –heat source;
2 –refrigerator; 3 –energy device

The case of two bodies is characterized by specific relations between the temperatures of the heat source and the refrigerator. The study of these relations is the subject of the following exposition.

2. Finding the relations between the temperatures of the two liquids. The parameters of both liquids are related by the elementary relations

$$G' dH' = -q(T', T'')\, dF, \tag{2,1}$$

$$G'' dH'' = \pm q(T', T'') \frac{T''}{T'}\, dF\,^{*}, \tag{2,2}$$

* The upper sign refers to concurrent flow, the lower to counterflow. In both cases the origin of the coordinate coincides with the place at which the hot liquid is supplied. The efficiency of conversion is equal to the Carnot efficiency.

G', H', G'', H'' are the flow rates and enthalpies, respectively, of the hot and cold liquids.

The heat q, removed from the heat carrier per unit time from a unit area of the heat-exchange surface F, may be any finite continuous* function of the temperatures T' and T'' satisfying the requirement

$$\operatorname{sign} q = \operatorname{sign}(T' - T''). \tag{2,3}$$

From (2,1) and (2,2), elementary relations are obtained for the connection between the initial and final temperatures:

$$G' \int_{T'f}^{T'_0} \frac{dH'}{T'} = G'' \int_{T''_0}^{T''_f} \frac{dH''}{T''}\,^{**} \tag{2,4}$$

or

$$G'(S'_0 - S'_f) = G''(S''_f - S''_0), \tag{2,5}$$

where H', H'', S', S'' are known functions of the corresponding temperatures.

Fig. 2. a–cocurrent flow, b–countercurrent flow: **1**–source of heat carrier; **2**–receiving reservoir of heat carrier; **3**–source of refrigerant; **4**–receiving reservoir of refrigerant; **5**–energy device

Fig. 2. a—cocurrent flow, b—countercurrent flow: 1—source of heat carrier; 2—receiving reservoir of heat carrier; 3—source of refrigerant; 4—receiving reservoir of refrigerant; 5—energy device

Figure 1: Fig. 2. a—cocurrent flow, b—countercurrent flow: 1—source of heat carrier; 2—receiving reservoir of heat carrier; 3—source of refrigerant; 4—receiving reservoir of refrigerant; 5—energy device

Expressions (2,4) and (2,5) are written in the same way for cocurrent and countercurrent flow of the liquid. However, between these cases there is a fundamental difference in the behavior of the final temperatures of both liquids when the heat-exchange surface is increased without bound ($F \to \infty$).

In the first case, the temperatures of both liquids tend to a common limit

$$\lim_{F \to \infty} T'_f = \lim_{F \to \infty} T''_f = T_c,$$

which is found from the solution of the equations

$$G'(S'_0 - S'_c) = G''(S''_c - S''_0). \qquad (2,6)$$

In the second case, the limiting temperature of that liquid for which the maximum possible change in the entropy flux $G[S_{T'_0} - S_{T''_0}]$ has the smaller value tends to the initial temperature of the other liquid. This property of countercurrent flow is easily proved by considering the finite integral

$$Q = \lim_{F \to \infty} \int q \, dF$$

with (2,3) and (2,5) satisfied.

* The first derivatives must also satisfy the continuity condition.
** In this form, (2,4) is valid for processes in which the enthalpy does not depend on pressure.

The maximum work $L_{\max}$ performed in the system in cocurrent flow will, naturally, occur when $T'_f = T''_f = T_c$. Its value is found by means of the expression

$$L_{\max} = G'(H'_0 - H'_c) - G''(H''_c - H''_0). \qquad (2,7)$$

It is easy to show that in the case of counterflow the maximum work will likewise occur when $T'_f = T''_f = T_c$, and will be determined by the same relation (2,7).

Indeed, if in a counterflow system the hot liquid is overcooled and the cold liquid overheated ($T''_f > T_c > T'_f$), then, over the existing temperature difference, additional work can be obtained by means of a continuous sequence of direct

Carnot cycles. The entire work capacity of the system will be realized when the temperature difference is fully utilized.

The difference between cocurrent flow and counterflow from the energy standpoint is that in the first case the maximum work is realized as $F \to \infty$, whereas in the second case it is realized at a finite surface F_{opt}, for which $T_f' = T_f'' = T_c$.

An increase in the heat-exchange surface (beyond the value F_{opt}), leading to overheating of the cold liquid and overcooling of the hot one, causes a decrease in the work capacity of the system. Thermodynamically, such an increase in the magnitude of F is equivalent to including refrigeration Carnot cycles in the system with maximum work capacity.

3. Finding analytical relations between temperatures for the case when $\Delta H = c\Delta T$ (with $c = \text{const}$). The solution of equations (2,1) and (2,2) leads under these conditions to the following relations between temperatures: for the case of cocurrent flow,

$$T_0'/T' = (T''/T_0'')^{\mu} \tag{3,1}$$

and for counterflow,

$$T_0'/T' = (T_f''/T'')^{\mu}. \tag{3,2}$$

The maximum work capacity for both cases is represented by the expression

$$L_{\max} = G'c'T_0' \left[1 + \mu a - a^{\mu/(\mu+1)}(\mu+1)\right] * \tag{3,3}$$

Here and in the other expressions the indices 0 and f refer, respectively, to the inlet and outlet sections; c is the heat capacity.

The value of the temperature $T_c = T_f' = T_f''$ is found by means of

$$T_c = T_0' a^{\mu/(\mu+1)}. \tag{3,4}$$

In expressions (3,1), (3,2), (3,3), and (3,4), $\mu = G''c''/G'c'$, $a = T_0''/T_0'$. In the case of counterflow, as $F \to \infty$, the final temperature of the liquid whose flow heat capacity Gc has the smaller value tends to the initial temperature of the other liquid.

Hence, for $\mu > 1.0$ the expressions $\lim_{F\to\infty} T_f' = T_0''$ and $\lim_{F\to\infty} T_f'' = T_0' a^{\mu+1/\mu}$ are valid; for $\mu < 1.0$, $\lim_{F\to\infty} T_f' = T_0' a^{\mu}$ and $\lim_{F\to\infty} T_f'' = T_0'$. It is easy to show that for $\mu = 1.0$, $\lim_{F\to\infty} T_f' = T_0''$ and $\lim_{F\to\infty} T_f'' = T_0'$, and, as a consequence, $\lim_{F\to\infty} L = 0$.

4. The simplest method for accounting for cycle imperfection. It is of some interest to analyze those cases in which, within a certain temperature range, the efficiency of an elementary cycle can be represented—

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* Equation (2,3) may also be used in calculating the maximum work for the case of nonstationary exchange between immobile reservoirs of heat and cold (Fig. 1). The quantities G' and G'' in this case are the masses of the corresponding reservoirs.

is represented by the expression: $\eta = \eta_0(T' - T'')/T'$, where η_0 is the relative efficiency. The heat-balance equations for the two fluids will be written, respectively, as

$$G'c'dT' = -q\,dF, \tag{4,1}$$

$$G''c''dT'' = \pm\,[1 - \eta_0(1 - T''/T')]\,dF. \tag{4,2}$$

The integrals of (4,1), (4,2) for the co-current and counter-current cases, respectively, are written as

$$\frac{T'}{T'_0} = \left[\frac{(1-\eta_0) + (\eta_0 + \mu)a}{(1-\eta_0) + (\eta_0 + \mu)T''/T'}\right]^{\mu/(\mu+\eta_0)}, \tag{4,3}$$

$$\frac{T'}{T'_0} = \left[\frac{(1-\eta_0) + (\eta_0 - \mu)T''_f/T'_0}{(1-\eta_0) + (\eta_0 - \mu)T''/T'}\right]^{\mu/(\mu-\eta_0)}. \tag{4,4}$$

The limiting value of the common final temperature corresponding to the maximum work for the co-current case is written as

$$\frac{T'_f}{T'_0} = \left[\frac{(1-\eta_0) + (\eta_0 + \mu)a}{(1-\eta_0) + (\eta_0 + \mu)}\right]^{\mu/(\mu+\eta_0)}. \tag{4,5}$$

The value $y = T'/T'_0$, corresponding to the maximum work for the counter-current case, is found by using the relation between $y = T'/T'_0$ and $x = T''/T'_0$, defined by (4,4), and by finding the extremum of the expression $L = G'c'T'_0[1 + \mu a - (\mu x + y)]$ for the work.

Use of these dependences leads to an irrational equation for y

$$\eta_0 a y^{-(\eta_0+\mu)/\mu} + (1 - \eta_0)y^{-\eta_0/\mu} = 1.0. \tag{4,6}$$

The considerations stated earlier concerning the temperatures of the heat carrier and refrigerant corresponding to maximum works in co-current and counter-current flows at an efficiency equal to the efficiency of the Carnot cycle are not applicable in the present case.*

Conclusions. 1. The limiting maximum work capacities for the cases of co-current and counter-current flow of the hot and cold fluids are equal to one another. However, in the case of counter-current flow, realization of the limiting regime is achieved on a heat-exchange surface of finite size.

2. The regime of maximum work capacity for both flow cases is characterized by equality of the fluid temperatures in the outlet cross sections. An increase in the heat-exchange surface (the counter-current case), leading to supercooling of the hot fluid and superheating of the cold fluid (as compared with the regime of equal temperatures in the outlet cross sections), leads to a decrease in the work capacity of the system.
3. In the case of counter-current flow, with an unlimited increase in the heat-exchange surface, the final temperature of the fluid whose flow heat capacity (or maximum increase in entropy flow) has the smaller value tends to the initial temperature of the other fluid.

The author expresses his gratitude to Ya. B. Zeldovich for discussion of the work and valuable advice.

Received
20 I 1967

CITED LITERATURE

1. J. W. Gibbs, *Trans. Connecticut Acad.*, **3**, pp. 108, 345 (1875–1878); Collected Works of J. W. Gibbs, Thermodynamic Works, Moscow–Leningrad, 1950.

* For example, in the counter-current case the optimum temperatures of the hot and cold fluids are not equal to one another and have values higher than those calculated from (3,4).

Note: Figure translations are in progress. See original paper for figures.