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Abstract

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ON THE RELATION BETWEEN BOHM' S CRITERIA AND THE GENERATION OF "AMBIPOLAR" SOUND IN THE PLASMA OF THE POSITIVE COLUMN OF A GAS DISCHARGE

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Bohm' s stability criterion for the wall layer ([1]) (the layer in which there is a substantial deviation from quasineutrality), without allowing for the possible distribution of ions over velocities near the plasma boundary (starting from which a quasineutral treatment is already applicable), has the form

$$\Phi_0 \equiv -e\varphi_0/M > 1/2 \quad \text{or} \quad v_{ir} > \sqrt{T_e/M}; \tag{1}$$

φ_0 is the potential of the plasma boundary relative to the discharge axis; v_{ir} is the ion velocity at the plasma boundary, equal to $\sqrt{2e\varphi_0/M}$.

If condition (1) is not fulfilled, the potential of the wall layer, as Bohm showed, begins to oscillate, and the solutions outside and inside the wall layer do not match. In this sense the "stability of the wall layer" is understood ([1,3]). In deriving (1), ionization and collisions in the wall layer were not taken into account; it was assumed that the ions are unmagnetized and freely fly apart in the electric field of the layer toward the wall, while the electrons are distributed according to Boltzmann.

In ([2]) it was shown that, when the distribution of ions over velocities at the plasma boundary is taken into account, Bohm' s criterion has the form

$$\overline{v_{ir}^2} > T_e/M, \tag{2}$$

where $\overline{v_{ir}^2}$ is the mean value of the square of the drift velocity of ions at the column boundary.

Regarding diffusion in the plasma of the positive column as ambipolar and taking into account that $v_{ir}^2 \approx v_{\text{A}\Gamma}^2$ ($v_{\text{A}\Gamma}$ is the velocity of the ambipolar flux at the column boundary), one may say that criteria (1), (2) are satisfied when the velocity of ambipolar diffusion of the carriers at the column boundary exceeds the velocity of ion sound:

$$v_{\mathrm{A\Gamma}} > \sqrt{T_e/M}. \tag{3}$$

Criterion (3) can be violated in a sufficiently strong magnetic field $\omega_{eH}\tau_e \gg 1$, although still $\omega_{iH}\tau_i \ll 1$, where $\omega_{e,iH}$, $\tau_{e,i}$ are, respectively, the cyclotron frequencies and relaxation times of the carriers, determined by collisions with neutrals, since the velocity of ambipolar diffusion decreases with increasing magnetic field. This was first shown by Hsu ([3]), who carried out a calculation* for a high-pressure positive column (diffusion regime: $\lambda_{i,e} \ll a$). However, Hsu' s attempt to apply the result obtained to interpret the known experiments of Lehnert ([4]) proved unsuccessful, since Hsu' s paper contains no information on perturbations developing in the unstable regime. The theoret–

* In deriving this criterion, Hsu used not entirely justified assumptions in choosing the position of the plasma boundary relative to the boundary determined by the Schottky condition.

A physical interpretation of Lenert' s instability was given by Kadomtsev and Nedospasov [(5)], who developed the theory of the helical current-convective instability of the plasma of a high-pressure positive column in the presence of a constant magnetic field parallel to the column axis. The excitation of this instability is associated with the longitudinal electric field that maintains the required level of ionization in the stationary state.

Below we shall attempt to relate Bohm' s criterion to the criterion for the generation of "ambipolar" sound [(6)]—purely azimuthal waves with a frequency of the order of the inverse time for ion loss in the ambipolar electric field to the tube wall, whose excitation mechanism is not connected with the longitudinal electric field.

Since the rate of ambipolar diffusion of the carriers determines the rate at which perturbations are carried out of the plasma, it is natural to assume that, when (3) is not satisfied, the diffusion loss does not have time to damp transverse perturbations with phase velocity $\sim \sqrt{T_e/M}$, which is expressed in the appearance of spatial oscillations of the potential of the near-wall layer, leading to choking of the loss to the wall. Under these conditions the cord as a whole is displaced toward the wall, which corresponds to the instability of the mode $m = 1$, $k_z = 0$. The frequency of the oscillations corresponding to this mode is of the order of the inverse time of ambipolar diffusion.

In [(6)] a detailed calculation was carried out of the stability of the positive-column plasma at low pressure under conditions in which the process of free ion loss in the ambipolar electric field is dominant ($\lambda_i, \rho_{iH} > a$; $\rho_{eH} \ll a$, where ρ_H are the Larmor radii of the particles). It was shown that this process, in combination with the drift motion of the electrons, leads to the buildup of a purely azimuthal wave (mode $m = 1$, $k_z = 0$) with a frequency of the order of the inverse time of ion loss in the ambipolar field:

$$\omega \approx \frac{2}{a}\sqrt{e\varphi/M}. \tag{4}$$

Taking into account that at the stability boundary $e\varphi_0 \approx T_e$,

$$\omega \approx \frac{2}{a}\sqrt{\frac{T_e}{M}} \approx 2.2 Z_{\text{cr}}, \tag{5}$$

where Z_{cr} is the number of ionizations produced by one electron per unit time in the critical state.

As we have already mentioned, these waves were called by us "ambipolar" sound. The criterion for excitation of these waves (formula 3.18 [(6)]) has the form

$$\omega_{eH}^2 \geq 1.8 M^{1/2} T_e / m_e^{3/2} \lambda_e a. \tag{6}$$

Both in frequency and in the excitation criterion, the waves observed under the indicated conditions in the experiments of Nedospasov et al. [(7)] agree satisfactorily with the predictions of the theory of ambipolar sound.

The condition for violation of Bohm' s criterion (3) in such a system approximately coincides with condition (6). Indeed, the ambipolar diffusion velocity in such systems ($\lambda_i > a$) is

$$v_{\text{Ar}} \approx \frac{D_e}{(\omega_H \tau)_e^2} \frac{\nabla n}{n} \approx \frac{D_e}{a(\omega_H \tau)_e^2}. \tag{7}$$

The condition for violation of (3) (taking into account that $D_e = \tau_e T_e / m_e$, $v_{eT} = \sqrt{3T_e/m_e}$)

$$\omega_{eH}^2 > 1.7 M^{1/2} T_e / m_e^{3/2} \lambda_e a \tag{8}$$

practically coincides with (6).

In this connection we assumed that the criterion for violation of (3) corresponds to the excitation in the plasma of the positive column of azimuthal waves with a frequency of the order of the inverse time of ambipolar diffusion (ambipolar sound).

We shall use this conclusion in calculating the criterion for excitation of ambipolar sound in the plasma of the positive column at high pressure. The conditions for excitation of these waves now arise already in the layer near the wall of thickness λ_i (larger than the Debye radius), in which the ions are unmagnetized:

$$\lambda_i \gg \sqrt{T_e/4\pi n e^2}, \qquad (\omega_H \tau)_i \ll 1. \tag{9}$$

The electrons in this layer drift, since

$$\rho_{eH} \ll \lambda_i \sim \lambda_e, \qquad (\omega_H \tau)_e \gg 1. \tag{10}$$

A rigorous calculation of stability, carried out in work ([6]) for a low-pressure plasma, when the role of such a layer is played by the entire plasma, is in this case extremely difficult, since the dispersion equation can be obtained only by matching solutions from the layer, where the processes of ion transport in the ambipolar electric field dominate, and from the column, where the diffusion regime dominates.

As we have already mentioned, Bohm' s criterion for the positive column at high pressure in a magnetic field was written down by Hsu ([3]). In that work, the choice of the plasma boundary relative to the boundary determined by Schottky' s condition (zero density at the wall) was made from the condition of best agreement of the resulting criterion for stability of the near-wall layer with Lenert' s experimental results ([4]), which, in our view, is quite unjustified.

In our calculations the plasma boundary is chosen at a point one mean free path away from the boundary determined by Schottky' s condition, since the processes of free transport of ions in the ambipolar electric field begin to dominate over diffusion precisely at this point. The density and ambipolar-potential distributions derived from Schottky' s condition are likewise valid only up to this point, since beyond it the diffusion regime is violated.

Let us give the expressions for these distributions, derived for the case

$$(\omega_H \tau)_e \gg 1, \qquad (\omega_H \tau)_i \ll 1, \qquad T_e \gg T_i :$$

$$n_0(r) = N_0 J_0(\beta_0 r), \qquad \frac{d\varphi_0}{dr} = \frac{D_e}{b_e + b_i(\omega_H \tau)_e^2} \frac{1}{n_0} \frac{dn_0}{dr}, \qquad Z = \frac{D_e \dfrac{b_i}{b_e} \beta_0^2}{1 + \dfrac{b_i}{b_e}(\omega_H \tau)_e^2}, \tag{11}$$

where $\beta_0 a$ is the first zero of the Bessel function J_0.

The velocity of the ambipolar flux at the boundary of the column is

$$v_{Ar} = -b_i \nabla\varphi\big|_r = \frac{b_{iD} e}{b_e + b_i(\omega_H \tau)_e^2} \left.\frac{\beta_0 J_1(\beta_0 r)}{J_0(\beta_0 r)}\right|_{r=a-\lambda_i}. \tag{12}$$

Taking into account that $\lambda_i \ll a$:

$$J_0[\beta_0(a-\lambda_i)] \approx \beta_0\lambda_i J_1(\beta_0 a), \qquad J_1[\beta_0(a-\lambda_i)] \approx J_1(\beta_0 a),$$

$$v_{Ar} \approx \frac{b_{iD}e}{b_e + b_i(\omega_H\tau)_e^2}\,\frac{1}{\lambda_i}. \tag{13}$$

Since

$$D_e = \frac{T_e}{e}b_e, \qquad \lambda_i = \sqrt{\frac{3T_i}{m_i}}\,\tau_i = \frac{m_i}{e}\,b_i\sqrt{\frac{3T_i}{m_i}}, \tag{14}$$

the criterion for violation of (3) will have the form

$$(\omega_H\tau)_e^2 > \left(\sqrt{\frac{T_e}{3T_i}} - 1\right)\frac{b_e}{b_i} \qquad (T_e \gg T_i). \tag{15}$$

In deriving (15) we neglected the potential drop over the last mean free path, since this drop is

$$\approx \frac{\lambda_i}{a}\frac{T_e}{e} \ll \frac{T_e}{e}.$$

The frequency of the ambipolar sound at the stability boundary in this case is

$$\omega \approx 2.2 Z_{\rm cr} \approx 2.2\,\frac{D_e\dfrac{b_i}{b_e}\beta_0^2}{1 + \dfrac{b_i}{b_e}(\omega_H\tau)_{\rm cr}^2} \approx 3.8\sqrt{\frac{T_i}{T_e}\,D_e\,\frac{b_i}{b_e}\,\beta_0^2}. \tag{16}$$

The diffusion coefficient accompanying the excitation of ambipolar sound practically coincides with the Bohm coefficient and is equal to

$$D \approx C\,\frac{E_\varphi}{H}\,a \approx C\,\frac{\varphi_0 a}{2\pi a H} = \frac{cT_e}{4\pi e H}. \tag{17}$$

For helium, for example ([4,8]), at $p \sim 1$ mm Hg, $a \sim 1$ cm, $T_i \sim 10^{3^\circ}$, and T_e at the moment of instability $\sim 2.5 \cdot 10^{4\circ}$; $b_i \approx 3 \cdot 10^6$ CGSE units, $b_e/b_i \approx 44$. In this case $H_{\rm cr} \approx 2$ kOe, $f \approx 15$ kHz, $D \approx 10^4$ cm^2/sec. The critical magnetic field for the excitation of ambipolar sound is somewhat higher than the critical field for excitation of the helical instability under these conditions ($H_{\ell\rm cr} \approx 1.5$ kOe), while the frequency is somewhat lower ($f_\ell \approx 20$ kHz).

Since the excitation of these waves is associated with the process of ion acceleration in the ambipolar electric field over the last free path length, one should expect their amplitude to be maximal in the wall layer of thickness $\lambda_i \ll a$.

Thus, it is possible that the experimentally well-studied Lehnert helical instability is accompanied by a purely azimuthal satellite, generated at somewhat higher magnetic fields and at a somewhat lower frequency under conditions in which the ambipolar electric field has not yet changed its sign.

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Note: Figure translations are in progress. See original paper for figures.