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MATHEMATICS

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Abstract

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MATHEMATICS

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ON A CERTAIN CLASS OF PROBLEMS IN INTEGRAL GEOMETRY

(Presented by Academician I. M. Vinogradov on 10 VI 1967)

Let $\Gamma(x_1, x_2)$ be a two-parameter family of curves in the half-plane $y > 0$, satisfying the following conditions:

1) Through any pair of points of the half-plane with coordinates (x_1, y_1), (x_2, y_2), $x_1 \neq x_2$, there passes one and only one curve of the family. Moreover, through the pair of points $(x_1, 0)$, $(x_2, 0)$ there passes the curve $\Gamma(x_1, x_2)$.
2) If $x_1' < x_1 < x_2 < x_2'$, then the part of the half-plane bounded by the curve $\Gamma(x_1, x_2)$ lies strictly inside the part of the half-plane bounded by the curve $\Gamma(x_1', x_2')$.
3) The functions defining the equations of the curves Γ in normal form are three times continuously differentiable with respect to arc length and to the parameters x_1, x_2.

Further, let $u(x, y)$ be a function continuous in the half-plane, and let

$$v(x_1, x_2) = \int_{\Gamma(x_1, x_2)} u(x, y)\, ds.$$

Consider the following problem of integral geometry: given the function $v(x_1, x_2)$, it is required to determine the function $u(x, y)$.

A number of inverse problems for partial differential equations reduce to the formulated problem of integral geometry (see ([3],[4])). In the work of V. G. Romanov ([4]), analogous problems of integral geometry were investigated by the method of moments.

In the present note we give a formulation and a scheme of proof of the uniqueness theorem for the posed problem, without those additional restrictions on the families of curves Γ that are contained in ([5]).

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Theorem. *The function $u(x,y)$ is uniquely determined in the part of the half-plane $y>0$ bounded by the curve $\Gamma(x_1^0,x_2^0)$, $x_1^0<x_2^0$, by the values of the function $v(x_1,x_2)$ in the square $x_1^0\le x_1\le x_2^0$, $x_1^0\le x_2\le x_2^0$.*

We give the scheme of the proof of the theorem. Consider some point (x,y) lying on the curve $\Gamma(x_1,x_2)$, and denote by θ the angle formed by the tangent to the curve $\Gamma(x_1,x_2)$ at the point (x,y) with the x-axis, $x_1<x_2$. Denote by $\widetilde{\Gamma}(x,y,\theta)$ the part of the curve $\Gamma(x_1,x_2)$ lying between the points $(x_1,0)$, (x,y), and consider the function

$$w(x,y,\theta)=\int_{\widetilde{\Gamma}(x,y,\theta)} u(\xi,\eta)\,ds.$$

It is not difficult to verify that the function w satisfies the following second-order differential equation:

$$\frac{\partial}{\partial\theta}\left(\cos\theta\,\frac{\partial}{\partial x}+\sin\theta\,\frac{\partial}{\partial y}-K\,\frac{\partial}{\partial\theta}\right)w=0, \tag{1}$$

where $K(x,y,\theta)$ is the curvature of the curve $\Gamma(x_1,x_2)$ at the point (x,y).

The differential operator in equation (1) can be brought to the form

$$\begin{gathered}\left(K\partial/\partial\theta-\frac{1}{2}\cos\theta\,\partial/\partial x-\frac{1}{2}\sin\theta\,\partial/\partial y\right)^2-\\ -\frac{1}{4}\left(\cos\theta\,\partial/\partial x+\sin\theta\,\partial/\partial y\right)^2+\\ +K\left(\sin\theta\,\partial/\partial x-\cos\theta\,\partial/\partial y\right)+\left(\cos\theta K_x+\sin\theta K_y\right)\partial/\partial\theta.\end{gathered} \tag{2}$$

Let us consider, instead of the variables x,y,θ, new variables t,α,β such that t,α,β, as functions of x,y,θ, satisfy the equations

$$K\,\partial t/\partial\theta-\frac{1}{2}\cos\theta\,\partial t/\partial x-\frac{1}{2}\sin\theta\,\partial t/\partial y=0,$$

$$\cos\theta\,\partial\alpha/\partial x+\sin\theta\,\partial\alpha/\partial y=0,$$

$$\cos\theta\,\partial\beta/\partial x+\sin\theta\,\partial\beta/\partial y=0.$$

In the variables t,α,β, equation (1) takes the form:

$$a_0\partial^2w/\partial t^2-\left(a_1\partial/\partial\alpha+a_2\partial/\partial\beta\right)^2w+a_3\,\partial w/\partial t+$$

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$$+a_4\, \partial w/\partial \alpha + a_5\, \partial w/\partial \beta = 0, \tag{3}$$

where the a_j are certain functions.

With a suitable choice of the variables t, α, β, the integral-geometry problem under consideration is reduced to the Cauchy problem for equation (3). The Cauchy data are prescribed on the surface

$$t = \varphi(\alpha, \beta),$$

where φ is a function with positive second differential.

The formulated uniqueness theorem follows, after the transformations indicated above, from Hörmander' s general theorem (see (^{2})).

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