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Abstract

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HYDROMECHANICS

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ASYMPTOTIC LAW OF PROPAGATION OF A PLANE DETONATION WAVE

The laws of degeneration of plane shock waves propagating through a gas at rest are well known. If t is time and r is the coordinate of the wave, then the asymptotic law of propagation of the wave has the form ([1])

$$a_1(t - t_0) = r\left(1 - \sqrt{\frac{r_0}{r}} - \frac{1}{8}\frac{r_0}{r}\ln\frac{r}{r_0} + \cdots\right). \tag{1}$$

Here a_1 is the speed of sound in the undisturbed gas; t_0 and r_0 are certain constants. The shock wave asymptotically degenerates into a sound wave, and the disturbed flow behind it becomes a Riemann traveling wave.

Let us study the asymptotic behavior of a plane strong detonation wave in the case where the disturbed motion behind the wave weakens it and turns into a Chapman–Jouguet wave. Let v, p, and ρ be the velocity, pressure, and density of the gas, and let c be the velocity of the detonation wave. We shall denote by the subscript 1 the pressure and density in the undisturbed gas.

The relations on the detonation wave can be written in the form

$$-\rho_1 c = \rho(v - c),$$

$$\rho_1 c^2 + p_1 = \rho(v - c)^2 + p,$$

$$\frac{1}{2}c^2 + \frac{a_1^2}{\gamma - 1} + Q = \frac{1}{2}(v - c)^2 + \frac{\gamma}{\gamma - 1}\frac{p}{\rho}.$$

Here Q is the heat supplied to a unit mass of gas.

From these relations one can obtain the following expressions for the velocity, pressure, and density of the gas behind the wave:

$$v = \frac{a_1}{\gamma + 1}\frac{1 - q}{\sqrt{q}} + \frac{a_1}{\gamma + 1}\frac{\sqrt{1 - q_{jq}}}{\sqrt{q}}\sqrt{1 - \frac{q}{q_j}}, \tag{2}$$

$$p = p_1 + \frac{\rho_1 a_1 v}{\sqrt{q}}, \qquad \rho = \frac{\rho_1}{1 - v\sqrt{q}/a_1}.$$

Here $q = a_1^2/c^2$; q_j is the value of q corresponding to Chapman–Jouguet detonation and determined by the formula

$$\frac{a_1}{\gamma+1}\frac{1-q_j}{\sqrt{q_j}} = \sqrt{2\frac{\gamma-1}{\gamma+1}Q}\,.$$

If $Q = 0$, then $q_j = 1$, and expression (2) turns into the usual formula for an adiabatic shock wave

$$v = \frac{2a_1}{\gamma+1}\frac{1-q}{\sqrt{q}}.$$

Consider a detonation wave whose intensity exceeds only slightly the intensity of the Chapman–Jouguet wave. For such a wave the quantity $\varepsilon = \sqrt{1 - q/q_j}$ is small. Rewrite expression (2) for v and the corresponding expressions for p and ρ in the form

$$v = \frac{c_j}{\gamma+1}(1-\varepsilon^2)^{-1/2}\left(1 - q_j + q_j\varepsilon^2 + \varepsilon\sqrt{1 - q_j^2 + q_j^2\varepsilon^2}\right),$$

$$\frac{p}{p_1} = 1 + \frac{\gamma c_j v}{a_1^2}(1-\varepsilon^2)^{-1/2}, \qquad \frac{\rho_1}{\rho} = 1 - \frac{v}{c_j}(1-\varepsilon^2)^{1/2}.$$

Using the fact that ε is small, one can expand the right-hand sides of these expressions in powers of ε (assuming here that the Chapman-Jouguet wave is sufficiently strong, i.e., the quantity q_j is not close to unity). As a result, retaining terms of order ε^2, we obtain

$$\begin{aligned}
\frac{v}{v_j} &= 1 + \sqrt{\frac{1+q_j}{1-q_j}}\,\varepsilon + \frac{1+q_j}{2(1-q_j)}\,\varepsilon^2 + \cdots,\\
\frac{p}{p_j} &= 1 + \gamma\frac{v_j}{a_j}\left(\sqrt{\frac{1+q_j}{1-q_j}}\,\varepsilon + \frac{1}{1-q_j}\,\varepsilon^2 + \cdots\right),\\
\frac{\rho_j}{\rho} &= 1 - \frac{v_j}{a_j}\left(\sqrt{\frac{1+q_j}{1-q_j}}\,\varepsilon + \frac{1}{1-q_j}\,\varepsilon^2 + \cdots\right).
\end{aligned} \tag{3}$$

Here v_j, p_j, ρ_j—the values of the velocity, pressure, and density of the gas behind the Chapman–Jouguet wave—are determined by the formulas

$$v_j = \frac{c_j}{\gamma+1}(1-q_j), \qquad p_j = p_1 + \rho_1 c_j v_j, \qquad \rho_j = \rho_1 \frac{c_j}{c_j - v_j}.$$

From relations (3), in particular, it follows that

$$\frac{p}{\rho^\gamma} = \frac{p_j}{\rho_j^\gamma}\left[1 + \frac{\gamma(\gamma-1)}{2}\left(\frac{1-q_j}{\gamma+q_j}\right)^2 \varepsilon^2 + O(\varepsilon^3)\right],$$

$$a - \frac{\gamma-1}{2}v = a_j - \frac{\gamma-1}{2}v_j + \frac{1}{4}v_j\left[\gamma - 1 - \frac{(\gamma+1)^2}{2}\frac{1+q_j}{\gamma+q_j}\right]\varepsilon^2 + O(\varepsilon^3).$$

Thus, with accuracy up to and including terms of order ε, the gas parameters behind the detonation wave satisfy the relations of a Riemann traveling wave (we note that, in the case of an ordinary shock wave, the expressions $\frac{p}{\rho^\gamma}$ and $a - \frac{\gamma-1}{2}v$ contain only even powers of ε and are constants with accuracy up to and including terms of order ε^4).

In the general case, from the relations on the detonation wave and the equations of motion one can obtain the following expression for the derivative $\partial v/\partial r$ immediately behind the discontinuity:

$$\frac{\partial v}{\partial r} = \frac{2(1+q) + \sqrt{1-q_j q}\sqrt{1-q/q_j}}{2(1-q_j q)\left(1 - \dfrac{q}{q_j}\right)}\frac{v}{q}\frac{dq}{dr}. \tag{4}$$

For a Riemann traveling wave,

$$v = \Phi[r - (a+v)t], \qquad a - \frac{\gamma-1}{2}v = a_j - \frac{\gamma-1}{2}v_j,$$

where Φ is an arbitrary function, with respect to which we shall assume that $r\Phi'(\xi) \to \infty$ as $r \to \infty$ and $\xi \to \xi_0$, where ξ_0 is the limiting value of $\xi = r - (a+v)t$ on the detonation wave. From the expression written above for v, it is easy to obtain

$$\frac{\partial v}{\partial r} = \frac{\Phi'(\xi)}{1 + \dfrac{\gamma+1}{2}t\Phi'(\xi)}.$$

As $r \to \infty$,

$$\frac{\partial v}{\partial r} \to \frac{2}{\gamma+1}\frac{c_j}{r}.$$

Substituting this expression into the left-hand side of relation (4) and retaining on its right-hand side only the leading terms in ε, we obtain the asymptotic equation

$$\frac{2}{r} = -\frac{1}{\varepsilon^2}\frac{d\varepsilon^2}{dr}.$$

Hence we find the asymptotic law of propagation of the detonation wave

$$c_j(t-t_0) = r\left(1+\frac{r_0^2}{r^2}+\cdots\right). \tag{5}$$

The values of the gas-dynamic quantities behind the detonation front are expressed by the asymptotic formulas

$$\frac{v}{v_j} = 1+\sqrt{2\frac{1+q_j}{1-q_j}\frac{r_0}{r}}+\cdots, \qquad \frac{p}{p_j} = 1+\gamma\frac{v_j}{a_j}\sqrt{2\frac{1+q_j}{1-q_j}\frac{r_0}{r}}+\cdots,$$

$$\frac{\rho_j}{\rho} = 1-\frac{v_j}{a_j}\sqrt{2\frac{1+q_j}{1-q_j}\frac{r_0}{r}}+\cdots$$

The asymptotic behavior of the detonation wave described by formula (5) differs essentially from the asymptotic behavior of an ordinary shock wave.

According to relation (1), the shock wave in the t, r plane has no asymptote, receding ahead to an infinite distance from any straight line $r - a_{jt} = \text{const}$. In contrast, the detonation wave tends to a definite asymptote $r - c_{jt} = \text{const}$.

In conclusion, we note that in a completely analogous way it is easy to obtain the asymptotic form of a detonation wave in an established plane gas flow.

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CITED LITERATURE

1. L. I. Sedov, *Similarity and Dimensional Methods in Mechanics*, Moscow, 1957.

Note: Figure translations are in progress. See original paper for figures.