

VIBRATIONS OF AN UNBOUNDED LAYERED PLATE IN A GAS FLOW

THEORY OF ELASTICITY

1967

SovietRxiv

View the original and related papers at <https://sovietrxiv.org/items/ru-196701.90509>

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

Abstract

Full Text

UDC 533.6.0.113.42

THEORY OF ELASTICITY

A. I. SMIRNOV

VIBRATIONS OF AN UNBOUNDED LAYERED PLATE IN A GAS FLOW

(Presented by Academician L. I. Sedov on 6 IV 1966)

§ 1. The system of differential equations for flexural vibrations of a plate can be reduced to a single equation with respect to the displacement function $\chi(x, t)$ of the form (1)

$$(1 - \vartheta h^2 \beta^{-1} \nabla^2) \nabla^2 \nabla^2 (\chi(x, t)) + \frac{\Omega}{D} \frac{\partial^2}{\partial t^2} (1 - h^2 \beta^{-1} \nabla^2) \chi(x, t) = \frac{q(x, t)}{D}, \quad (1, 1)$$

where $\nabla^2 = \partial^2 / \partial x^2$; $\vartheta, h, \beta, \Omega, D$ are constants depending on the elastic, geometrical, and mass characteristics of the three-layer beam (for details see (1)); $q(x, t)$ is the specific aerodynamic load on the plate.

The deflection $w(x, t)$ is related to the displacement function by the relation

$$w(x, t) = (1 - h^2 \beta^{-1} \nabla^2) \chi(x, t). \quad (1, 2)$$

Let us determine the possibility of the occurrence of transverse vibrations of the form

$$\chi(x, t) = \chi_0 e^{i\alpha(ct-x)}, \quad (1, 3)$$

where χ_0 is some constant (real or complex); α is the wave number; c is the phase velocity of propagation of the elastic wave in the plate; x is the longitudinal coordinate. By virtue of the linearity of the problem, we take

$$|\alpha \chi_0| = |2\pi \chi_0 / \lambda| \ll 1, \quad (1, 4)$$

where $\lambda = 2\pi / \alpha$ is the wavelength.

Since the solution (1,3) must be finite as $x \rightarrow \pm\infty$, we restrict ourselves to discussion of the case of real wave numbers. For the phase velocity c the inequality

$$\operatorname{Im}(\alpha c) \leq 0 \quad (1, 5)$$

will hold. Otherwise $\chi(x, t)$ would grow without bound as $t \rightarrow -\infty$. The quantity $\operatorname{Im}(\alpha c)$ determines the character of the behavior of elastic waves with the passage of time. Thus, if solutions of equation (1,1) in the form (1,3) exist, then small vibrations of the panel will be divergent.

§ 2. Let us determine the aerodynamic load on the panel. We shall assume that the panel is flowed over on both sides by a potential flow of an ideal gas with identical physicommechanical characteristics. The bounded solution of the equation for the velocity potential $\Phi(x, y, t)$ (y is the transverse coordinate) in this case will be

$$\Phi(x, y, t) = A \exp[i\alpha(ct - x) - \alpha\gamma|y|], \quad (2, 1)$$

where

$$\gamma = \sqrt{1 - (V - c)^2/a^2}, \quad \operatorname{Re}(\alpha\gamma) > 0, \quad (2, 2)$$

A is a constant to be determined (a is the speed of sound in the undisturbed flow).

Taking (1.6) into account, we obtain that the radiation condition reduces to the requirement that the inequalities be satisfied jointly

$$\operatorname{Im} \gamma \geq 0, \quad \operatorname{Re} c \leq 0. \quad (2.3)$$

The boundary condition of impermeability of the plate takes the form

$$(\partial\Phi/\partial y)_{y=\pm 0} = (\partial w/\partial t + V \partial w/\partial x)_{y=\pm 0}, \quad (2.4)$$

where V is the velocity of the undisturbed gas flow directed along the x -axis.

Substituting (2.1), (1.2), and (1.3) into (2.4), we obtain

$$A = -i\chi_0(1 + h^2\beta^{-1}\alpha^2)\gamma^{-1}(c - V). \quad (2.5)$$

Consequently,

$$\Phi(x, y, t) = -i\chi_0(1 + h^2\beta^{-1}\alpha^2)\gamma^{-1}(c - V) \exp[i\alpha(ct - x) - \alpha\gamma|y|]. \quad (2.6)$$

The specific aerodynamic load on the plate is equal to

$$q(x, t) = -2\rho(\partial\Phi/\partial t + V \partial\Phi/\partial x)_{y=+0}. \quad (2.7)$$

Substituting (2.6), we find

$$q(x, t) = 2\chi_0\rho\gamma^{-1}(c - V)^2(1 + h^2\beta^{-1}\alpha^2) \exp i\alpha(ct - x). \quad (2.8)$$

§ 3. Substituting (2.8) into (1.1), we obtain

$$c^2 + \mu(c - V)^2 - c_0^2 = 0, \quad (3.1)$$

where $\mu = 2\rho/\Omega\alpha$ is the coefficient of added mass; c_0 is the phase velocity of an elastic wave in vacuum,

$$c_0 = \frac{D}{\Omega l^2} \frac{1 + \vartheta k\alpha^2}{1 + k\alpha^2} \alpha^2. \quad (3.2)$$

Thus, the problem reduces to investigating the behavior of the roots of the function of the complex variable c , given in the form (3.1). If, in a given range of variation of the parameters, there is at least one root of (3.1), then the amplitude of the panel oscillations will grow without bound with time.

A. Incompressible flow. Since equation (3.1) coincides in form with the analogous equation obtained in (3) for a homogeneous plate, the case of incompressible flow can be analyzed by the method presented there.

B. Compressible flow. Introduce the new variable $\zeta = (c - V)/V$. Equation (3.1) will be

$$f(\zeta) = (\zeta + 1)^2 + \mu\gamma^{-1}\zeta - \zeta_0^2 = 0, \quad (3.3)$$

where

$$\zeta_0 = c_0/V, \quad \gamma = \sqrt{1 - M^2\zeta^2}, \quad M = V/a. \quad (3.4)$$

In constructing the boundary separating the regions of stable and divergent oscillations, we shall use the idea of I. A. Vyshnegradskii^(4,5). Introduce the plane of parameters $z = \xi + i\eta$, $\xi = \mu$, $\eta = \zeta_0^2$, and write (3.3) in the form

$$f(\zeta, z) = P(\zeta)\xi + Q(\zeta)\eta - R(\zeta) = 0, \quad (3.5)$$

where

$$\begin{aligned}
 P(\zeta) &= \zeta^2 / \sqrt{1 - M^2 \zeta^2} = u_1(x, y) + iv_1(x, y), \\
 Q(\zeta) &= -1 = u_2(x, y) + iv_2(x, y), \\
 R(\zeta) &= -(\zeta + 1)^2 = u_3(x, y) + iv_3(x, y).
 \end{aligned}
 \tag{3.6}$$

Equation (3.5) is equivalent to the following two equations

$$\begin{aligned}
 u_3(x, y) &= u_1(x, y)\xi + u_2(x, y)\eta, \\
 v_3(x, y) &= v_1(x, y)\xi + v_2(x, y)\eta.
 \end{aligned}
 \tag{3.7}$$

The system (3.7) establishes a correspondence between the mappings $f(\zeta, z)$ in the ζ - and z -planes. Solving (3.7) with respect to ξ and η , we obtain

$$\xi = \frac{u_3 v_2 - v_3 u_2}{\Delta}, \quad \eta = \frac{u_1 v_3 - v_1 u_3}{\Delta},
 \tag{3.8}$$

where

$$\Delta = \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}.
 \tag{3.9}$$

Obviously,

$$\begin{aligned}
 u_1(x, y) &= \frac{x^2 - y^2}{r} \cos \varphi + \frac{2xy}{r} \sin \varphi, \\
 v_1(x, y) &= \frac{2xy}{r} \cos \varphi - \frac{x^2 - y^2}{r} \sin \varphi,
 \end{aligned}
 \tag{3.10}$$

where

$$\begin{aligned}
 r &= \{[1 - M^2(x^2 - y^2)]^2 + (2M^2xy)^2\}^{1/4}, \\
 \varphi &= \frac{1}{2} \operatorname{arc\,tg} \frac{-2M^2xy}{1 - M^2(x^2 - y^2)}.
 \end{aligned}
 \tag{3.11}$$

Further,

$$u_3(x, y) = -[(x + 1)^2 - y^2], \quad v_3(x, y) = -2(x + 1)y.
 \tag{3.12}$$

By virtue of (1.5) and (2.3) ($a > 0$), the zeros of the function $f(\zeta)$ that are of interest to us lie in the lower half-plane. The image of the real axis of the physical

plane Γ on the plane of the parameters z is, obviously, the line separating the regions of stable and divergent oscillations (the flutter boundary).

To find the parametric equations of Γ , we pass in expressions (3.11) to the limit as $y \rightarrow 0$ and, taking into account relations (3.8), (3.9), find the parametric equations of the required flutter boundary

$$\xi = -\frac{2(x+1)\sqrt{1-M^2x^2}}{(2+M^2x^2)x},$$

$$\eta = \frac{2+M^2x^2(x+1)}{2+M^2x^2}(x+1). \quad (3.13)$$

Fig. 1. Flutter boundaries of an unbounded three-layer strip for various Mach numbers

Obviously, $|x| \leq 1/M$, since otherwise the quantity ξ would be imaginary, which would contradict the physical meaning. Moreover, $\xi = \mu > 0$. Consequently, for the flutter boundary in the ζ -plane one must have $x < 0$.

If it is taken into account that as $x \rightarrow 0$, $\xi \rightarrow \infty$, $\eta \rightarrow 1$, then the flutter boundary in the plane of the parameters ξ, η is the image of the segment of the real axis x , located between $x_1 = -1/M$ and $x_2 = 0$. Let us establish the orienta-

image. If ⁽⁵⁾

$$\Delta > 0, \quad (3.14)$$

then the mapping preserves orientation. In the opposite case the orientation of the image is changed to the opposite one. From (3.13) we have

$$\Delta = 2 + M^2x^2 > 0. \quad (3.15)$$

Thus, the orientation of the image and of the original will be the same. Moving along the x -axis in the ζ -plane from the point $x = -1/M$ to $x = 0$ and leaving to the right the region of unstable oscillations, we shall once describe in the ξ, η -plane the flutter boundary, to the right of which there will be the region of divergent oscillations, and to the left the region of stable oscillations (Fig. 1). Taking (3.14) into account, we obtain ⁽⁵⁾ that, on passing from the upper half-plane ζ into the lower one (or conversely), or, in other words, on crossing the flutter boundary from left to right, the function $f(\zeta)$ acquires (or loses) one zero. Using the argument principle, one can show ⁽³⁾ that $f(\zeta)$ either has two zeros on the real axis and no zeros in the lower half-plane, or has a single zero in the lower half-plane. Thus, when the point ζ crosses the segment $-1/M \leq x \leq 0$,

passing from the upper half-plane into the lower one, $f(\zeta)$ acquires the zero indicated above and the flutter regime sets in.

For $M = 0$, formulas (3.13) pass into the corresponding formulas of work ⁽³⁾. The computations were carried out on the BESM-2 electronic computer.

The author expresses gratitude to E. I. Grigolyuk for posing the problem.

All-Union Institute of Scientific and
Technical Information

Received
16 III 1966

REFERENCES

- ¹ E. I. Grigolyuk, P. P. Chulkov, *Izv. AN SSSR, Mekh. i mashinostr.*, No. 1 (1964).
- ² E. I. Grigolyuk, A. P. Mikhailov, *DAN*, 158, No. 3 (1964).
- ³ J. W. Miles, *J. Aeronaut. Sci.*, 23, No. 8 (1956).
- ⁴ N. G. Chebotarev, N. N. Meiman, *Tr. Matem. inst. im. V. A. Steklova AN SSSR* (1949).
- ⁵ M. A. Lavrent'ev, B. V. Shabat, *Methods of the Theory of Functions of a Complex Variable*, Moscow, 1965.

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.