The theory of characteristic vectors and its application to the study of the asymptotic behavior of solutions of differential systems. II

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Abstract

The study of characteristic vectors, initiated in the author' s previous article, is continued. The concept of a superior vector is introduced, which is used to investigate the distribution of characteristic vectors of linear differential systems under small perturbations. This leads to stability criteria for characteristic vectors that generalize well-known results on the stability of characteristic exponents. At the end of the article, the superior vector is applied to study the stability of nonlinear systems. Bibliography: 5 items.

Full Text

Preamble

In this section, we consider the properties of solutions for the system of differential equations under various constraints. We assume that for 1967, III, No. 10, the condition $\bar{b} \ll$ MATH_0001$a < 2$ holds. Furthermore, we assume the condition $p_i - p_n > 0$ for $H > H_1$ $(i = 1, \dots, N)$, where $P_n = \int p(i) di$. Let the coefficients be defined as $a(m) = p(m)$ for $i = 1, \dots, k$. We define the matrix $X(t, \tau) = [x_{ij}(t, \tau)]$, where the components satisfy the exponential bound $x_{ii} = \delta_{ij} \exp \int p_i(g) dg$.

Given the initial conditions and the structure of the system, the solution $X(t, \tau)$ satisfies the relation $a(m)\{x_i(t, \tau)\} = a(m)$. We observe that the norm $\|x(t, \tau)\| < N$ is bounded by the exponential term $\exp \int p_{ii}(\lambda) d\lambda$. Following the methodology established in [**?**], we analyze the transition from equation (2.2) to (2.3) using the transformation $z = re^{-\alpha t}$ for $\alpha > 0$ and $B = \text{const}$. The fundamental solution matrix $X(t, \tau)$ is defined such that $X(\tau, \tau) = E$, with the components x_{rs} determined by the integral of the pressure or density functions

p_{ij}. Specifically, for $r > s$, the components are given by:

$$x_{rs} = \left(\exp \int p_{rr} d\sigma\right) \int \left(p_{rs} \exp - \int p_{ss} d\sigma\right) d\sigma$$

while for $r < s$, we have $x_{rs} = 0$ as per (2.4) and (2.5).

The asymptotic behavior of these solutions is characterized by the parameters $a(m)$ and $\delta(m)$. As shown in (8.5), the growth is bounded by $C_k D(\epsilon) e^{alphat}$, where the coefficients are determined by the vectors $\{x_{sl}\}$. The relationship between the spectral components and the stability of the system is further explored through the differential operator $L = \frac{d}{dt} p_i(m) + \delta(m)$. For the system defined in (0.2), we establish that the perturbations $\epsilon(m)$ remain within bounds that ensure the stability of the null solution.

1. Stability and Bounds

We consider the non-homogeneous system $Lx = A(t)x + f(t, x)$ as defined in (9.1). Here, the matrix $B(t) = [b_{ij}(t)]$ satisfies the norm condition $\|B(t)\| < \|I(t)\|^2$. The stability of the equilibrium point $x = 0$ is contingent upon the behavior of the characteristic exponents $a(m)$ and the perturbation terms $\delta(m)$. If the conditions in (9.1) and (9.2) are satisfied, the solution $x(t)$ remains bounded for all $t > t_0$.

Specifically, we utilize the integral representation for the solution $x_s(t)$:

$$x_s(t) = \sum_{k=1}^{n} c_k x_{sk}(t) + \int_{t_0}^{t} \sum_{\alpha=1}^{n} x_{s\alpha}(t, \tau) f_\alpha(\tau, x_1, \dots, x_n) d\tau$$

where the coefficients c_k are determined by the initial conditions at t_0. Using the estimates from (11.1) and (11.2), we can show that the norm of the solution satisfies $\|x(t)\| < 2k(\sum c_j^2)^{1/2} e^{a_0 t}$. This bound is critical for proving the existence of solutions in the limit as $t \to \infty$.

2. Nonlinear Perturbations and Asymptotic Behavior

For the nonlinear system (9.1) with the condition $\|f(t, x)\| < C(t)\|x\|^{1+\alpha}$ where $\alpha > 0$, we investigate the stability of the trivial solution $x = 0$. Suppose that the coefficients $C(t)$ are continuous for $t > t_0$ and satisfy the following conditions: - a) $a(m-1) = \beta$ and $a(m) < \beta$; - b) $a_k |C(t)| < K$ for some constant K.

Under these assumptions, the solution to the system (12.1) satisfies the stability criteria. By applying the transformation $x(t) = e^{-U(t)} y(t)$ as shown in (12.3), we reduce the system to a form where the asymptotic behavior of $y(t)$ can be analyzed via the Lyapunov method. The resulting system (12.4) allows us to establish that for any $\epsilon > 0$, there exists a neighborhood of the origin such that solutions starting within this neighborhood remain bounded by:

$$\|x(t)\| < N(\epsilon, Q)(\ln m_{t-1}) K$$

This result confirms that the spectral properties of the linear part, combined with the growth restrictions on the nonlinear term $f(t, x)$, guarantee the stability of the system. The findings are consistent with the classical results of Lyapunov and subsequent developments in the theory of differential equations [**?**, **?**].

Note: Figure translations are in progress. See original paper for figures.

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