PULSATION OF A ROTATING PLASMA CYLINDER AT ZERO TEMPERATURE GRADIENT

HYDROMECHANICS

1967

SovietRxiv

Abstract

Full Text

UDC 521.11+621.3.013

HYDROMECHANICS

E. V. RYAZANOV

PULSATION OF A ROTATING PLASMA CYLINDER AT ZERO TEMPERATURE GRADIENT

(Presented by Academician L. I. Sedov on 1 X 1966)

1. A number of works have been devoted to the solution of the problem of pulsation of a plasma cylinder (see, for example, [1–8]); in paper [1] an approximate solution of this problem was given, while in papers [2–8] certain classes of exact solutions describing periodic motions were obtained. In papers [3–5,7,8] the forces of gravity were taken into account; in papers [6,7] it was assumed that, owing to strong heat exchange between particles, instead of the adiabaticity condition the condition*

$$\frac{\partial T}{\partial r} = 0 \qquad (T \text{ —temperature}); \tag{1}$$

is satisfied; in paper [8] it was assumed that the gas particles rotate about the axis of symmetry with velocity v_φ.

All these classes of exact solutions were obtained under the assumption that the radial velocity v_r of a particle is a linear function of its distance r from the axis of symmetry, i.e.

$$v_r = \frac{r}{\mu(t)} \frac{d\mu}{dt}, \tag{2}$$

where $\mu(t)$ is some function of time. Motions of this type (with a linear dependence of the velocity on the distance) were first considered by L. I. Sedov [9,10].

In the present paper we consider one-dimensional axisymmetric unsteady motions of an electrically conducting perfect gas under the action of Newtonian gravitational forces.

We shall assume that the conductivity of the gas is infinite, viscosity is absent, the temperature does not depend on the geometrical coordinate but depends only on time, and the transverse component of the velocity $v_\varphi \neq 0$.

2. The equations describing such motions may be written in the form

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}-\frac{v_\varphi^2}{r}\right)+\frac{\partial}{\partial r}(p+h_\varphi+h_z)+\frac{2}{r}(h_\varphi+Gm\rho)=0,$$

$$\frac{\partial v_\varphi^2}{\partial t}+v_r\frac{\partial v_\varphi^2}{\partial r}+\frac{2}{r}v_r v_\varphi^2=0,\qquad \frac{\partial\rho}{\partial t}+v_r\frac{\partial\rho}{\partial r}+\rho\left(\frac{\partial v_r}{\partial r}+\frac{v_r}{r}\right)=0,\qquad (3)$$

$$\frac{\partial h_\varphi}{\partial t}+v_r\frac{\partial h_\varphi}{\partial r}+2h_\varphi\frac{\partial v_r}{\partial r}=0,\qquad \frac{\partial h_z}{\partial t}+v_r\frac{\partial h_z}{\partial r}+2h_z\left(\frac{\partial v_r}{\partial r}+\frac{v_r}{r}\right)=0,$$

$$\frac{\partial T}{\partial r}=0,\qquad \frac{\partial m}{\partial r}=2\pi\rho r\qquad \left(m=2\pi\int_0^\xi \eta\rho_1(\eta)\,d\eta\right).$$

Here ρ is the density; p is the pressure; m is the mass; G is the gravitational constant; $h_\varphi=\frac{1}{8\pi}H_\varphi^2$; $h_z=\frac{1}{8\pi}H_z^2$; H_φ and H_z are the transverse and axial components of the field-strength vector; $\xi=r/\mu$ is the initial radius; $\rho_1(\xi)$ is the initial density.

---

* In particular, condition (1) holds for isothermal flows.

By direct verification it is easy to see that system (3) has particular exact solutions of the following types:

I.

$$\rho=\frac{R'(\xi)}{r\mu},\qquad p=\frac{R'(\xi)}{r\mu}\varphi(\mu),\qquad v_\varphi^2=\frac{\xi^2\chi'(\xi)}{\mu^2R'(\xi)},$$

$$h_\varphi=\frac{1}{r^2}\{a_3+a_2[\xi^2R(\xi)-2\Pi(\xi)]-2\pi GR^2(\xi)\},\qquad (4)$$

$$h_z=\frac{\chi(\xi)+a_4R(\xi)}{\mu^4},\qquad m=2\pi R(\xi).$$

Here $\chi(\xi)$ is an arbitrary function of the Lagrangian coordinate; the dependence of the function R on ξ is determined by the formula $R'(\xi)=a_6\xi\exp(a_1\xi^2)$ (the prime denotes differentiation with respect to ξ); the function $\Pi(\xi)$ is related to $R(\xi)$ by the formula $\Pi'(\xi)=\xi R(\xi)$; $\varphi(\mu)$ is an arbitrary function; the dependence $\mu(t)$ is found from the differential equation

$$(d\mu/dt)^2=a_4\mu^{-2}-2a_2\ln\mu-4a_1F(\mu)+a_5=f_1(\mu),\qquad (4')$$

where

$$F(\mu)=\int\varphi(\mu)\mu^{-1}\,d\mu,\qquad a_i\ (i=1,2,\ldots,6)$$

are arbitrary constants.

II.

$$\rho = \frac{R'(\xi)}{r\mu}, \qquad p = \frac{R'(\xi)}{r\mu} b_1, \qquad v_\varphi^2 = \frac{\xi^2 \chi'(\xi)}{\mu^2 R'(\xi)},$$

$$h_\varphi = \frac{1}{r^2}\{b_3 + b_2[\xi^2 R(\xi) - 2\Pi(\xi)] - 2\pi G R^2(\xi) - b_1[\xi R'(\xi) - 2R(\xi)]\}, \quad (5)$$

$$h_z = \frac{\chi(\xi) + b_4 R(\xi)}{\mu^4}, \qquad m = 2\pi R(\xi).$$

Here $R(\xi)$, $\chi(\xi)$ are arbitrary functions, $\Pi'(\xi) = \xi R(\xi)$; the dependence $\mu(t)$ is determined from the equation

$$(d\mu/dt)^2 = b_4 \mu^{-2} - 2b_2 \ln \mu + b_5 = f_2(\mu), \quad (5')$$

b_i $(i = 1, 2, \dots, 5)$ are arbitrary constants.

III.

$$\rho = \frac{R'(\xi)}{r\mu}, \qquad p = \frac{R'(\xi)}{r\mu^3} c_1, \qquad v_\varphi^2 = \frac{\xi^2 \chi'(\xi)}{\mu^2 R'(\xi)},$$

$$h_\varphi = \frac{1}{r^2}\{c_3 + c_2[\xi^2 R(\xi) - 2\Pi(\xi)] - 2\pi G R^2(\xi)\}, \quad (6)$$

$$h_z = \frac{\chi(\xi) + c_4 R(\xi) - c_1 \xi^{-1} R'(\xi)}{\mu^4}, \qquad m = 2\pi R(\xi).$$

Here $R(\xi)$, $\chi(\xi)$ are arbitrary functions; $\Pi'(\xi) = \xi R(\xi)$; the equation used to determine the function $\mu(t)$ is

$$(d\mu/dt)^2 = c_4 \mu^{-2} - 2c_2 \ln \mu + c_5 = f_3(\mu), \quad (6')$$

where c_i $(i = 1, 2, \dots, 5)$ are arbitrary constants.

IV.

$$\rho = \frac{R'(\xi)}{r\mu}, \qquad p = \frac{R'(\xi)}{r\mu^3}(A_1\mu^2 + A_6), \qquad v_\varphi^2 = \frac{\xi^2 \chi'(\xi)}{\mu^2 R'(\xi)},$$

$$h_\varphi = \frac{1}{r^2}\{A_3 + A_2[\xi^2 R(\xi) - 2\Pi(\xi)] - 2\pi G R^2(\xi) - A_1[\xi R'(\xi) - 2R(\xi)]\}, \quad (7)$$

$$h_z = \frac{\chi(\xi) + A_4 R(\xi) - A_6 \xi^{-1} R'(\xi)}{\mu^4}, \qquad m = 2\pi R(\xi).$$

Here $R(\xi)$, $\chi(\xi)$ are arbitrary functions; $\Pi'(\xi) = \xi R(\xi)$; the form of the function $\mu(t)$ is determined by the equation

$$(d\mu/dt)^2 = A_4 \mu^{-2} - 2A_2 \ln \mu + A_5 = f_4(\mu), \quad (7')$$

where A_i $(i = 1, 2, \dots, 6)$ are arbitrary constants.

We note that solution IV includes, as special cases, solutions II and III.

If in formulas (4)–(6) one sets everywhere $v_\varphi = 0$ (i.e., assumes that $\chi = \text{const}$), $G = 0$, then one obtains the solutions published in paper ([6]); if $v_\varphi = 0$, but

$G \neq 0$, then from (4)–(6) follow the solutions found earlier by the author ([7]). The possible types of gas motion are determined by the form of the functions $f_i(\mu)$ ($i = 1, 2, 3, 4$). A detailed study of these motions for functions of the form (5′)–(7′) was given in papers ([2–4]), where it was shown that, depending on the values of the coefficients entering these functions, gas particles, rotating about the axis of symmetry, may execute motions of the type of periodic pulsations, fly away from the axis of symmetry to infinity, collapse toward the axis, etc. The presence in the expression for $f_1(\mu)$ of an arbitrary function $F(\mu)$ shows that, in case I, the most arbitrary types of motion are possible.

The solutions found may be useful in studying problems of the motion of cosmic gas masses subject to magnetic fields and Newtonian gravitational forces.

In particular, as in papers ([2,8]), one can investigate radial pulsations of a gas cylinder of finite radius r_0, in which the gas particles also possess a transverse component of velocity. At $r = r_0$, on the surface of the cylinder, the relation

$$p_*(r_0, t) = \left(\rho + h_\varphi + h_z\right)\big|_{r=r_0}, \tag{8}$$

must be satisfied, expressing the condition of equality of the total pressures outside and inside the cylindrical column of gas.

Received
10 IX 1966

CITED LITERATURE

[1] M. A. Leontovich, S. M. Osovets, *Atomic Energy*, No. 3, 81 (1956).
[2] A. G. Kulikovskii, DAN, 114, No. 5, 984 (1957).
[3] I. M. Yavorskaya, DAN, 114, No. 5, 988 (1957).
[4] E. V. Ryazanov, PMM, 23, issue 1, 187 (1959).
[5] G. C. McVittie, *Rev. Modern Phys.*, 30, No. 3, 1080 (1958).
[6] V. P. Korobeinikov, E. V. Ryazanov, DAN, 124, No. 1, 51 (1959).
[7] E. V. Ryazanov, DAN, 126, No. 6, 1224 (1959).
[8] Yu. P. Ladikov, DAN, 137, No. 2, 303 (1961).
[9] L. I. Sedov, *Similarity and Dimensional Methods in Mechanics*, Moscow, 1961.
[10] L. I. Sedov, DAN, 90, No. 5, 735 (1953).

Note: Figure translations are in progress. See original paper for figures.