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Abstract

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ON THE "CORRECTION" OF FUNCTIONS OF SEVERAL VARIABLES DEFINED ON THE TORUS

(Presented by Academician S. N. Bernstein on 6 XII 1966)

The classical theorem of D. E. Menshov (see ([1]), and also ([2])) on the "correction" of functions states that any measurable, almost everywhere finite function $f(x)$ of period 2π can be corrected on a set of arbitrarily small measure so that the Fourier series of the "corrected" function converges uniformly.

It turns out that this result, which is a strengthening of N. N. Luzin's C-property, reflects the specificity not only of Fourier series as an approximation apparatus. In ([5]) it was shown that any measurable, finite almost everywhere function $f(x)$ can be changed on a set of small measure so that its Fourier series and Fourier-Walsh series converge uniformly to the corrected function. It can be shown that the theorem on correction of a function also holds for measurable functions of many variables defined on the torus, when they are expanded into a multiple Fourier series with respect to the trigonometric system.

The present note is devoted to the question of the correction of functions of many variables in connection with their expansion into a series with respect to the Walsh system (for the definition of the orthonormal Walsh system, as well as its properties, see ([4])).

It is known (see, for example, ([3])) that in passing from functions of one variable to functions of several variables and from simple series to multiple ones, in many cases one has to consider and essentially distinguish various kinds of convergence to the limit (ordinary convergence, restricted convergence, etc.). In connection with this, the question naturally arises whether it is necessary to consider one or another special kind of convergence in the problem of correction of functions of many variables. For definiteness we consider the typical case of functions of two variables.

Let $f(x, y)$ be a function of period 1 in each variable, integrable on the square of periods, and let

$$\frac{a_{00}}{4}+\frac{1}{2}\sum_{k=1}^{\infty}(a_{k0}\cos 2k\pi x+b_{k0}\sin 2k\pi x)+$$

$$+\frac{1}{2}\sum_{l=1}^{\infty}(a_{0l}\cos 2l\pi y+c_{0l}\sin 2l\pi y)+$$

$$+\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(a_{kl}\cos 2k\pi x\cdot\cos 2l\pi y+b_{kl}\sin 2k\pi x\cdot\cos 2l\pi y+$$

$$+c_{kl}\cos 2k\pi x\cdot\sin 2l\pi y+d_{kl}\sin 2k\pi x\cdot\sin 2l\pi y),$$

$$\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}c_{kl}\psi_k(x)\psi_l(y)$$

are its double Fourier and Fourier-Walsh series, respectively.

Denote by $S'_{mn}(f;x,y)$ and $S''_{mn}(f;x,y)$ the partial sums of order m in x and of order n in y, respectively, of the Fourier and Fourier—Walsh series of the function $f(x,y)$. Using the general scheme of reasoning given in ([2]), one can prove the following assertion.

Theorem 1. Let $f(x,y)$ be a measurable function, finite almost everywhere on $[0,1;0,1]$. For any $\varepsilon>0$, one can construct a function $g(x,y)$ coinciding with $f(x,y)$ on a set E, $mE>1-\varepsilon$, and such that the sequences of partial sums $S'_{mn}(g;x,y)$ and $S''_{mn}(g;x,y)$ of its Fourier and Fourier—Walsh series converge uniformly on $[0,1;0,1]$ to the function $g(x,y)$ as m and n tend to infinity arbitrarily.

If $S_{mn}(f;x,y)$ is the partial sum of order m in x and of order n in y of the multiple series that is Fourier in x and Fourier—Walsh in y for the function $f(x,y)$, then the following holds.

Theorem 2. Let $f(x,y)$ be a measurable function, finite almost everywhere on $[0,2;0,1]$. For any $\varepsilon>0$, one can construct a function $g(x,y)$ coinciding with $f(x,y)$ on a set E, $mE>2\pi-\varepsilon$, and such that the sequence of partial sums of its multiple series, Fourier in x and Fourier—Walsh in y, converges uniformly on $[0,2\pi;0,1]$ to the function $g(x,y)$ as m and n tend to infinity arbitrarily.

The propositions given above show that for functions of many variables D. E. Menshov' s theorem on correction is valid in its strongest version.

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.