TEMPERATURE GREEN' S FUNCTIONS

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Abstract

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FOR A WEAKLY NONIDEAL BOSE-EINSTEIN SYSTEM

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Earlier in ([1,2]), for a Bose system, a theory of temperature elementary excitations was constructed by analogy with the theory of elementary excitations for a Fermi system with long-range interaction ([3]); recently Hohenberg ([4]) and Havouré ([5]) have shown that at zero temperature $\theta = 0$ and for a small interaction there can be no special collective branch of the spectrum, the existence of which was discussed in ([1,2]). In the present work we give results obtained by the author which confirm ([4,5]) (within the approach he is developing to the theory of a Bose gas ([6])), as well as, apparently, some new results not known in the literature.

First of all, let us make a small remark concerning the simple secular equation (7) of ([1]), which is valid when exchange effects are neglected.* In this equation, in $Q_0(q;E)$ one may put $\varepsilon(p) = E(p)$, $A(p) = E(p)$, $B(p) = 0$, and obtain equation (15) of ([1]) without the second term. In ([1]) it was shown that such an equation has a phonon root as $v(0) \to \infty$; we shall show that the root is absent in the weak-coupling theory as $v(0) \to 0$.

Take the function

$$\mathrm{Re}(q,\varepsilon) = \frac{1}{(2\pi)^3}\int dp\,(n_p - n_{p+q})\frac{E(p+q)-E(p)}{-\varepsilon^2 + (E(p+q)-E(p))^2}; \tag{1}$$

where the integral should be understood in the sense of the principal value. For $\varepsilon = 0$ the function is positive and, for small q,

$$\mathrm{Re}(q,0) \simeq \frac{m^2\theta}{2\hbar^2|q|}; \tag{2}$$

as $\varepsilon \to \infty$ the function assumes negative values, moreover

$$\mathrm{Re}(q,\varepsilon) \simeq -\frac{2E(q)n}{\varepsilon^2}\left(\frac{\theta}{\theta_0}\right)^{3/2}; \tag{3}$$

at $\varepsilon = E(q) = \hbar^2 q^2/2m$ the function undergoes a discontinuity with jump, for small q,

$$\Delta R \simeq \frac{m^2\theta}{4\hbar^4}\frac{1}{|q|}. \tag{4}$$

As $q \to 0$, the value of the function on the left goes to infinity, while the value on the right tends to a certain negative limit,

$$\frac{1}{(2\pi)^2}\,\frac{2}{\theta}\,\frac{(2m\theta)^{3/2}}{\hbar^3}\int_0^{+\infty} x^2 dx\left(\frac{e^{x^2}}{(e^{x^2}-1)^2}-\frac{1}{x^4}\right); \tag{5}$$

the schematic behavior of $\mathrm{Re}(q,\varepsilon)$ for small q is presented in Fig. 1.

The corresponding secular equation can be represented in the form

$$\mathrm{Re}(q,\varepsilon) = -1/v(q); \tag{6}$$

* Exchange effects may be neglected for a small long-range interaction ([2]); for a small short-range interaction the exchange effects are important ([4,5]).

it is quite clear that (6) has no root as $v(0) \to 0$, in agreement with ([4,5]), where in fact $v(q) = v(0)$ is assumed. This means that there is no root for a small short-range interaction; for a small long-range interaction a root of the equation may exist.

Let us now consider the secular equation (4) from (2) in the case $\theta \neq 0$. We linearize it in Ξ, H, Γ, and Π; for Ξ, H we restrict ourselves to diagrams of second order, for Γ—to diagrams of first order, and for Π—to zeroth order. Using the expansions in small v for $A(p), B(p)$ at fixed p, and for $\Xi_2(p, E_{2k})$, $H_2(p, E_{2k})$, $\Gamma_1(p, E_{2k})$, and $\Pi_0(p, E_{2k})$ at fixed p, E_{2k}, we obtain the approximate equation

$$\begin{aligned} &E_{2k}^2 + E^2(p) + 2n_0 v(p)E(p) + 2E(p)\frac{1}{(2\pi\hbar)^3}\times \\ &\times\int dp'\,(v(p-p')-v(p'))n_{p'} + (E_{2k}^2 + E^2(p))\times \\ &\times\frac{v(p)}{(2\pi\hbar)^3}\int dp'\,\frac{n_{p-p'}-n_{p'}}{E(p')-E(p-p')+iE_{2k}} + \text{terms higher than } v = 0, \end{aligned} \tag{7}$$

Fig. 1. Schematic depiction of the behavior of the function $\mathrm{Re}(q,\varepsilon)$

which, when the fourth term is neglected, coincides with (15) of ([1]). Taking $p = \rho x$, $\rho = 2m^{1/2} n_0^{1/2} v^{1/2}(0)$, $E_{2k} = 2n_0 v(0) z$, i.e., p, E_{2k} are also small for small v, and using the expansions of $\Xi_2(p, E_{2k})$, $H_2(p, E_{2k})$, $\Gamma_1(p, E_{2k})$, and $\Pi_0(p, E_{2k})$ in small v at fixed x, z, we obtain the equation

$$
\begin{aligned}
&4n_0^2 v^2(0) z^2 + 4n_0^2 v^2(0) x^2 (1+x^2) + \frac{2}{\pi\hbar^3} m^{3/2} n_0^{3/2} v^{5/2}(0)\theta(x^2+1) + \\
&+ \frac{2}{\pi^3\hbar^3} m^{3/2} n_0^{3/2} v^{5/2}(0)\theta z^2 \int dx' \, \frac{1}{e(x')} \left(\frac{{x'}^2 - 1/2}{e(x')} + \frac{(x-x')^2 - 1/2}{e(x-x')} \right) \times \\
&\times \frac{1}{\left(e(x-x') + e(x')\right)^2 + z^2} - \frac{1}{\pi^3\hbar^3} m^{3/2} n_0^{3/2} v^{5/2}(0)\theta z^2 \int dx' \left(\frac{1}{E(x')} - \frac{1}{e(x-x')} \right) \times \\
&\times \left(\frac{{x'}^2 + 1/2}{e(x')} - \frac{(x-x')^2 + 1/2}{e(x-x')} \right) \frac{1}{\left(e(x') - e(x-x')\right)^2 + z^2} \\
&+ \frac{2}{\pi^3\hbar^3} m^{3/2} n_0^{3/2} v^{5/2}(0)\theta x^2 \int dx' \times \\
&\times \frac{e(x')e(x-x') - \frac{1}{2}({x'}^2 + (x-x')^2) - 3{x'}^2 (x-x')^2 - \frac{1}{4}}{e^2(x') e(x-x')} \frac{e(x') + e(x-x')}{\left(e(x') + e(x-x')\right)^2 + z^2} \\
&- \frac{1}{\pi^3\hbar^3} m^{3/2} n_0^{3/2} v^{5/2}(0)\theta x^2 \int dx' \, \frac{e(x')e(x-x') + \frac{1}{2}({x'}^2 + (x-x')^2) + 3{x'}^2 (x-x')^2 + \frac{1}{4}}{e^2(x') e^2(x-x')} \times \\
&\times \frac{\left(e(x') - e(x-x')\right)^2}{\left(e(x') - e(x-x')\right)^2 + z^2} - \frac{1}{\pi^3\hbar^3} m^{3/2} n_0^{3/2} v^{5/2}(0)\theta x^2 \times \\
&\times \int dx' \, \frac{e(x')e(x-x') + {x'}^2 (x-x')^2 + \frac{1}{2}({x'}^2 + (x-x')^2)}{e^2(x') e(x-x')} \frac{e(x-x') + e(x')}{\left(e(x-x') + e(x')\right)^2 + z^2} + \\
&+ \frac{1}{2\pi^3\hbar^3} m^{3/2} n_0^{3/2} v^{5/2}(0)\theta \int dx' \, \frac{e(x')e(x-x') - {x'}^2 (x-x')^2 - \frac{1}{2}({x'}^2 + (x-x')^2)}{e(x') e(x-x')} \times \\
&\times \frac{e(x') - e(x-x')}{\left(e(x') - e(x-x')\right)^2 + z^2} + \text{terms higher than } v^{5/2} = 0,
\end{aligned}
\tag{8}
$$

where $e(x) = x(x^2+1)^{1/2}$; in deriving (8), from the very beginning (4) from (2) was divided by $1 + \dfrac{v(q)}{V}\Pi$, which is equivalent to multiplying the linearized equation by $1 - \dfrac{v(q)}{V}\Pi$, i.e., to adding to it the term $\dfrac{v(q)}{V}\Pi(E_{2k}^2 + A^2 - B^2)$. In (8) the "gap" terms (which do not vanish at $x = 0$, $z = 0$) enter virtually; they compensate each other ([6]).

Putting in (8) $z^2 = -s^2 x^2$ and passing to the limit $x \to 0$, we obtain

$$4n_0^2 v^2(0)s^2 = 4n_0^2 v^2(0) - \frac{1}{3\pi\hbar^3} m^{3/2} n_0^{3/2} v^{5/2}(0)\theta -$$

$$-\frac{1}{\pi^2\hbar^3} m^{3/2} n_0^{3/2} v^{5/2}(0)\theta \int_0^\infty dx\, \frac{8x^6 + 12x^4 + \frac{13}{2}x^2 + 1}{(x^2+1)^2(x^2+\frac{1}{2})^2} \times$$

$$\times \left(2 + \frac{\sqrt{x^2+1}}{2x^2+1} \ln \frac{2x^2+1-\sqrt{x^2+1}}{2x^2+1+\sqrt{x^2+1}}\right) + \text{ terms higher than } v^{5/2}, \qquad (9)$$

whence for the velocity of sound we have

$$c = \frac{n^{1/2} v^{1/2}(0)}{m^{1/2}} + \frac{mv(0)\theta}{\hbar^3}\alpha + \text{ terms higher than } v^2, \qquad (10)$$

where α is a certain number, n is the total density. Using formulas (8) and (9) from (7) and the thermodynamic relation $mc^2 = n\, d\mu/dn$, one can obtain the "thermodynamic" velocity of sound

$$c = \frac{n^{1/2} v^{1/2}(0)}{m^{1/2}} - \frac{1}{4\pi\hbar^3} mv(0)\theta + \text{ terms higher than } v^2, \qquad (11)$$

which, however, does not coincide with the "microscopic" velocity of sound (10). From (8) one can also obtain the damping of elementary excitations

$$\Gamma(\rho x) = \frac{1}{8\pi\hbar^3} m^{3/2} n_0^{1/2} v^{3/2}(0)\theta(5\pi - 8)x + \text{ terms } v^{3/2} \text{ higher than } x + \text{ terms higher than } v^{3/2}, \qquad (12)$$

where $\rho = 2m^{1/2} n_0^{1/2} v^{1/2}(0)$; the damping at $\theta \neq 0$ is proportional to x, and not to x^5, as at $\theta = 0$.

We now consider equation (4) from (2) in the case $\theta = 0$. Proceeding similarly to the case $\theta \neq 0$, one can obtain for the velocity of sound

$$c = \frac{n^{1/2} v^{1/2}(0)}{m^{1/2}} - \frac{1}{8\pi^2\hbar^3} \frac{n^{1/2}}{m^{1/2} v^{1/2}(0)} \int_0^\infty p^2 dp\, \frac{v^2(p)}{E(p)} + \frac{1}{\pi^2\hbar^3} mnv^2(0) + \text{ terms higher than } v^2; \qquad (13)$$

using (6), (7) from (7) and the thermodynamic relation

$$m^2 c^2(n,\theta) = n\, \partial\mu(n,\theta)/\partial n,$$

one can obtain for c an expression exactly coinciding with (13). For the damping we obtain

$$\Gamma(\rho x) = x^5 \frac{3}{20\pi\hbar^3} m^{3/2} n_0^{3/2} v^{5/2}(0) + \text{ terms } v^{5/2} \text{ higher than } x^5 + \text{ terms higher than } v^3. \tag{14}$$

Formulas (13), (14) coincide with those obtained in the literature.

For the Green' s functions at $\theta = 0$ we obtain

$$TG^{-+} = -TG^{--} = n_0 v(0) - \frac{n_0}{4\pi^2\hbar^3} \int_0^\infty p^2 dp \, \frac{v^2(p)}{E(p)} + \frac{3}{\pi^2\hbar^3} m^{3/2} n_0^{3/2} v^{5/2}(0) +$$

$$+ \text{ terms } v^{5/2}, \text{ vanishing as } x \to 0 + \text{ terms higher than } v^{5/2}, \tag{15}$$

$$T = -s^2 x^2 \left(4n_0^2 v^2(0) + \frac{4}{\pi^2\hbar^3} m^{3/2} n^{5/2} v^{7/2}(0) \right) +$$

$$+x^2 \left(4n_0^2 v^2(0) - \frac{1}{\pi^2\hbar^3} n_0^2 v(0) \int_0^\infty {p'}^2 dp' \, \frac{v^2(p')}{E(p')} + \frac{40}{3\pi^2\hbar^3} m^{3/2} n_0^{5/2} v^{7/2}(0) \right) +$$

$$+ \text{ terms } v^2, \ v^3 \text{ and } v^{7/2} \text{ higher than } x^2 + \text{ terms higher than } v^{7/2}, \tag{16}$$

where it has been set that $E_{2k}^2 = -4n_0^2 v^2(0) s^2 x^2$ and $\rho = 2m^{1/2} n_0^{1/2} v^{1/2}(0) x$.

The result (15), (16) is consistent with the Gavoret relation[1]

$$G^{-+} \simeq \frac{n_0}{n} mc^2 \, \frac{1}{-E^2 + c^2 p^2}$$

which is valid for small E, p. In the approximation adopted, a similar relation also holds for G^{--}.

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[1]J. Gavoret, *Application de la théorie des perturbations à l' étude un liquide de Bose au zéro absolu*, Université de Paris, 1963.

References

Note: Figure translations are in progress. See original paper for figures.

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