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Abstract

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INVESTIGATION OF THE CRITICAL STATE OF A LIQUID BY MEANS OF GRAVITY WAVES

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Recently Makarevich carried out an interesting experiment ([1]). In a horizontal vessel of diameter ~ 1 cm and length ~ 20 cm there was sulfur hexafluoride SF_6 in a state close to critical. Along the liquid–vapor interface, by means of a stirrer moving in the horizontal direction along the vessel, a surface wave was excited.

At temperatures $(T_k - T)/T_k \sim 10^{-5}$ the penetration depth of the wave was of the order of millimeters, and the velocity was several centimeters per second. At temperatures $(T_k - T)/T_k \sim 10^{-3}$ a wave excited in the same way propagated with a higher velocity; the penetration depth, apparently, also increased.

The assumption that Makarevich observed gravity waves makes it possible to give a natural explanation of the experimental data. Indeed, the dispersion law for gravity waves on the interface of incompressible sufficiently deep liquids with densities ρ_1 and ρ_2 has the form ([2]):

$$\omega^2 = kg(\rho_1 - \rho_2)/(\rho_1 + \rho_2), \tag{1}$$

where the reciprocal wave vector k^{-1} determines the attenuation of the gravity wave into the depth of the liquid.

When the temperature changes from $(T_k - T)/T_k \sim 10^{-3}$ to $(T_k - T)/T_k \sim 10^{-5}$, the densities of the liquid and vapor approach each other by an order of magnitude (along the coexistence curve $(T_k - T)/T_k \sim [(\rho - \rho_k)/\rho_k]^2$), and for the given frequency of the excited wave (of the order of several hertz) the wave vector increases by an order of magnitude, i.e., the characteristic depth of attenuation of the gravity wave decreases.

For a more detailed investigation it is necessary first of all to take into account the compressibility of the liquid, which increases without bound as the critical point is approached ([3]) and may, generally speaking, turn out to be essential in the problem of gravity waves.

On approaching the critical point the heat capacities c_p and c_v increase without bound, i.e., the Péclet numbers become very large, and in the hydrodynamic equations thermal conductivity may be neglected. The growth of the heat capacities also leads to a shift toward lower frequencies of the boundary between adiabatic (low-frequency) and isothermal regimes of wave propagation; therefore, in what follows, in numerical estimates we shall use both the adiabatic and the isothermal speed of sound.

Let us first consider the problem of the propagation of gravity waves on the free surface of a compressible liquid.

The solution of the system of linearized hydrodynamic equations ([2]), depending on t and x in the form $\exp(-i\omega t+ikx)$ and decaying into the depth of the liquid ($z<0$), has the form

$$V_x = e^{-i\omega t+ikx}\left(Ae^{mz}+Be^{sz}\right), \qquad V_z = e^{-i\omega t+ikx}\left(-\frac{ik}{m}Ae^{mz}-\frac{is}{k}Be^{sz}\right),$$

$$\rho = e^{-i\omega t+ikx}\frac{\rho_0\omega Be^{sz}}{k\left[c^2-i\omega\left({}^4\!/_3\nu+\xi\right)\right]}-\frac{\rho_0 gz}{c^2}, \qquad m^2=k^2-\frac{i\omega}{\nu},$$

$$s^2=k^2-\frac{\omega^2}{c^2-i\omega\left({}^4\!/_3\nu+\xi\right)}. \tag{2}$$

Here $c^2=(\partial p/\partial\rho)_s$, or (see above) $(\partial p/\partial\rho)_T$; ν,ξ are the coefficients of shear and bulk viscosity; spatial dispersion (of the Ornstein–Zernike type ([2])) was not taken into account in the equation of state. The dispersion equation is obtained by substituting (2) into the boundary conditions at the surface of the liquid ($\sigma_{zz}=0,\ \sigma_{xz}=0$)

$$gs/\nu^2k^4+\left(2-i\omega/\nu k^2\right)^2=4ms/k^2. \tag{3}$$

Neglecting viscosity, we have from (3)

$$\omega^2=-\frac{g^2}{2c^2}\left[1\pm\left(1+\frac{4k^2c^4}{g^2}\right)^{1/2}\right]. \tag{4}$$

For $k>g/2c^2$, equation (3) describes a gravitational wave, $\omega^2\approx kg$, while for $k<g/2c^2$ it describes an ordinary sound wave, $\omega\approx kc$. The appearance of a sound branch of oscillations in a compressible liquid is physically quite obvious. It is not difficult to obtain from (3) also the damping law for gravitational waves.

Let us now turn to the problem of a gravitational wave propagating along the interface of two compressible liquids with densities ρ_1 and ρ_2 and viscosities ν_1

and ν_2. This case corresponds to the experiments mentioned above near the critical point of a liquid–vapor system, sufficiently close to the critical point, where the parameters of the liquid and vapor are very close.

The solution of the hydrodynamic equations in the upper liquid ($z > 0$) has the form (2), with the replacement $z \to -z$ and $V_z \to -V_z$. The dispersion equation is obtained by matching at the boundary the velocities V_x and V_z, as well as the components of the stress tensor σ_{xz} and σ_{zz}.

Table 1

Parameters of gravitational waves propagating along the boundary of two compressible liquids. The second and fourth columns were obtained from (11), the third from (12), the fifth from (7), and the sixth from (8)

	$\frac{T_k - T}{T_k}$	$\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$	$\frac{g}{c_s^2}\left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)$, cm^{-1}	$\frac{g}{2c_T^2}\left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)$, cm^{-1}	k, cm^{-1}	U, cm/sec	$\frac{g^2}{\omega^3}\left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)$, cm^2/sec
$\omega =$ 1 Hz	10^{-2}	10^{-1}	10^{-8}	10^{-3}	10^{-2}	50	10^{4}
$\omega =$ 1 Hz	10^{-4}	10^{-2}	10^{-7}	10^{-2}	10^{-1}	5	10^{2}
$\omega =$ 1 Hz	10^{-6}	10^{-3}	10^{-8}	10^{-1}	1	0.5	1
$\omega =$ 10 Hz	10^{-2}	10^{-1}	10^{-8}	10^{-3}	1	5	10
$\omega =$ 10 Hz	10^{-4}	10^{-2}	10^{-7}	10^{-2}	10	0.5	10^{-1}
$\omega =$ 10 Hz	10^{-6}	10^{-3}	10^{-8}	10^{-1}	10^{2}	0.05	10^{-3}

For the excitation of gravitational, and not sound, waves at the boundary of two liquids it is necessary that the condition be fulfilled (see Table 1):

$$k > \frac{g}{2c^2}\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}. \tag{5}$$

Neglecting the viscosity of the liquids, the dispersion equation differs from the case of an incompressible liquid by replacing k with $\chi = k(1 - \omega^2/k^2c^2)^{1/2}$, i.e., for liquids bounded above and below (at distances h_1 and h_2) by horizontal planes,

$$\omega^2 = \chi g \frac{\rho_1 - \rho_2}{\rho_1 \operatorname{cth} \chi h_1 + \rho_2 \operatorname{cth} \chi h_2}, \qquad \chi = k\left(1 - \frac{\omega^2}{k^2c^2}\right)^{1/2}, \tag{6}$$

and for deep liquids ($\chi h_1, \chi h_2 \gg 1$) and when condition (5) is satisfied,

$$\omega^2 = kg\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} - \frac{g^2}{2c^2}\left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)^2. \tag{7}$$

The velocity of propagation of the wave along the boundary of two sufficiently deep compressible liquids is

$$U = \frac{1}{2}\left(\frac{g}{k}\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)^{1/2}\left[1 + \frac{g}{kc^2}\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right]. \tag{8}$$

When the viscosity of the liquids is taken into account, periodic solutions will occur under the condition

$$\nu_{1,2} < \frac{\omega}{k^2} \simeq \frac{g^2}{\omega^3}\left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)^2. \tag{9}$$

The dispersion law in the first approximation in $\nu k^2/\omega$ has the form:

$$\omega^2 \simeq kg\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\left\{1 - 4\left[\left(\frac{i\nu_1 k^2}{\omega}\right)^{1/2} + \left(\frac{i\nu_2 k^2}{\omega}\right)^{1/2}\right] \times \right.$$
$$\left. \times \left[\left(1 + \sqrt{\frac{\nu_2}{\nu_1}}\right)\left(1 + \frac{\rho_1}{\rho_2}\right)\left(\frac{\rho_1}{\rho_2} + \sqrt{\frac{\nu_1}{\nu_2}}\right)\right]^{-1}\right\}. \tag{10}$$

Let us turn to numerical estimates. The values of the squares of the adiabatic c_s^2 and isothermal c_T^2 sound velocities, as well as the parameters of the coexistence curve–the relation between $(T_k - T)/T_k$ and $(\rho - \rho_k)/\rho_k$–can be taken from independent experiments. For an estimate we shall use dimensional considerations (typical parameters $p_k = 10$ atm and $\rho_k = 1$ g/cm^3), assuming, in accordance with the classical theory [2], that the compressibility is proportional to the distance to the critical point, and that the coexistence curve is a quadratic parabola:

$$c_T^2 = \left(\frac{\partial p}{\partial \rho}\right)_T \sim \frac{p_k}{\rho_k}\frac{T_k - T}{T_k} \sim 10^7\frac{T_k - T}{T_k}\ \frac{\text{cm}^2}{\text{sec}^2},$$

$$\frac{T_k - T}{T_k} \sim \frac{(\rho - \rho_k)^2}{\rho_k^2} \sim \left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)^2. \tag{11}$$

Far from the critical point the adiabatic speed of sound is $c_s \approx 10^5$ cm/sec, while sufficiently near the critical point it decreases as [4]

$$\frac{1}{c_v} \sim \frac{1}{\ln[(T_k - T)/T_k]},$$

i.e., in the crudest estimate,

$$c_s \sim 10^5 \text{ cm/sec} \quad \text{for} \quad (T_k - T)/T_k \sim 10^{-2};$$

$$c_s \sim 10^4 \text{ cm/sec} \quad \text{for} \quad (T_k - T)/T_k \sim 10^{-4} \text{ and } 10^{-6}. \tag{12}$$

Table 1 gives numerical estimates of the parameters of a gravitational wave propagating along the interface of two compressible liquids, for the characteristic frequencies $\omega = 1$ Hz and $\omega = 10$ Hz.

As the critical point is approached, the penetration depth of the wave and its propagation velocity decrease. At the same time, beginning with certain temperatures sufficiently close to the critical one, criterion (9) may be violated (if $\rho_1 - \rho_2 \to 0$ faster than $\nu_{1,2}$), and the motion becomes aperiodic.

The compressibility of the liquid is insignificant in the adiabatic regime, while in the isothermal case it may give corrections of up to 10%.

The determination of the parameters of the critical state—in particular, the density jump and the viscosity—by means of observations of gravitational waves appears promising: a number of quantities can be studied: the amplitu-

of sound, velocity, and damping, which, as is seen from Table 1, depend sharply on the temperature and frequency ranges.

All of our discussion, as well as the experiments mentioned above, is preliminary in character. For a detailed study and comparison with experiment it is necessary to take into account the methods of exciting and observing gravitational waves, the boundary conditions along the length of the vessel, the inclusion of nonlinear and dispersion effects in the hydrodynamic equations, the possible influence of capillarity, etc. Such work will be justified when special experiments are undertaken.

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Note: Figure translations are in progress. See original paper for figures.