APPLICATION OF ADJUSTABLE FEEDBACK FOR IDENTIFICATION AND ESTIMATION OF THE STABILITY MARGIN OF STOCHASTIC OBJECTS

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Abstract

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APPLICATION OF ADJUSTABLE FEEDBACK FOR IDENTIFICATION AND ESTIMATION OF THE STABILITY MARGIN OF STOCHASTIC OBJECTS

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An object with one input and one output is considered. It is known that uncontrolled stationary random disturbances $\zeta(t)$ enter the input of the object, as a result of which a random process $x(t)$ is present at the output. It is also known that the object itself, in the course of operation, generally undergoes continuous stationary random changes, as a result of which some of the objects of the given population may exhibit instability with respect to small disturbances (instability is understood in the sense of unbounded growth of the variance of the process $x(t)$ as $t \to \infty$; for other possible definitions of statistical instability see, for example, ([1])). It is required, on the basis of measurements of the process $x(t)$ (possibly applying a special signal $y(t)$ to the input of the object), to quantitatively characterize the degree of closeness of the object to the boundary of the stability region.

One of the possible ways of solving the problem consists in connecting to the object an adjustable feedback circuit, whose block diagram for the general case is shown in Fig. 1. The synthesis of this circuit is carried out in such a way that, by varying the parameters k, θ, it is possible to bring the closed system containing the object into a self-oscillatory regime; the amplitude of the self-oscillations is limited by an appropriate choice of the characteristic NE, shown in the figure.

Let us note that in the particular case when the object does not vary with time and $\zeta(t) \equiv 0$, the scheme described above makes it possible to obtain a solution of the stated problem without invoking any hypotheses at all concerning the structure of the object. In this case the boundary of the stability region is constructed in the space of the parameters of the feedback circuit; these parameters are then used as characteristics of the "stability margin." For example, in the case of inertia-free feedback and $\theta = 0$, from the value $k = k_*$ corresponding to the onset of self-oscillations, the "amplitude stability margin" is readily determined

Fig. 1. Block diagram of a controlled feedback circuit.

Figure 1: Fig. 1. Block diagram of a controlled feedback circuit.

(for the definition of this term see, for example, ([2])). Similar characteristics may be used, for example, for a comparative analysis of different objects.

The same method of estimating the "stability margin" is also applicable when $\zeta(t) \not\equiv 0$. In this case, however, in order to obtain statistical estimates of the boundary of the stability region, it is necessary to solve the problem of "recognizing" the narrow-band random process obtained when the process $\zeta(t)$ acts on a stable system, and the periodic process modulated by random noise (such a process is obtained when the closed system enters a self-oscillatory regime). A way of solving this problem, based on comparison of the distributions of the envelopes of the process $x(t)$, is indicated in the monograph ([3]).

In the general case the investigation of the object is carried out in the regime of established self-oscillations of the closed system (objects that have exhibited inherent instability during the tests are rejected, since–

that, within the framework of the adopted method, only stable objects can, generally speaking, be investigated). A hypothesis is adopted concerning the structure of the object (i.e., the form of the dependence between the processes $x(t)$ and $y(t)$ is specified). In the case under consideration, the requirement imposed on this model of the object is that it adequately describe the behavior of the process $x(t)$ when the object loses stability; therefore the parameters of the model must be random functions of time.

Next the problem is posed as follows: on the basis of measurements of the process $x(t)$, determine the statistical characteristics of the parameters of the model (the identification problem). Of primary interest are precisely those characteristics that determine whether the object with the feedback circuit disconnected will be stable or unstable; the corresponding boundary of the stability region in the space of statistical characteristics of the parameters of the object model can be found, for example, by simulation. The solution of the identification problem can also be used to estimate the reliability of the object in cases where other types of failures are possible (for example, when the value x reaches some maximum permissible value; here problems arise of determining the probability of reaching a dangerous state during a specified period of operation of the object). In addition, the problem of identifying stochastic systems is also of independent interest ([4]). Naturally, of course, to solve it in the general case it is necessary to introduce certain additional hypotheses concerning the statistical characteristics of randomly varying parameters.

Fig. 1. Block diagram of a controlled feedback circuit. O –object; LCh_1, LCh_2 –linear parts of the feedback circuit; U –amplifier with controllable gain coefficient k; BRZ –block of controllable delay $(z_2(t) = z_1(t-\theta))$; NE –nonlinear element

For models of objects in which the relation between the processes at the input and output is determined by a linear differential equation whose coefficients are stationary random functions of time, one can obtain an approximate analytical solution of the identification problem, starting from known methods for solving the corresponding "direct" problems of statistical dynamics ([3,5–7]). In ([4]) an iterative process is described for finding the moment functions of the coefficients of a linear system under a sinusoidal input signal. This solution is based on the assumption of relatively small random variations of the coefficients; the indicated assumption is verified in the process of practical computation of successive approximations. The same method can also be applied in the case under consideration, provided only that the nonlinearity is linearized (either by the method of harmonic linearization ([5]) in the case of a narrow-band process at the input of the NE, or by the method of statistical linearization ([5,6])); especially simple calculations are obtained for a "quasi-harmonic" system that gives a narrow-band process at the output ([3,6]).

The approximate analytical solutions found can also be used when solving the problem by direct selection of the statistical characteristics of the model parameters on an analog computer or a digital computer.

Let us consider as an example a model of the object in the form of a second-order system with a randomly varying damping coefficient. Let LCh_1 contain a differentiating element, and let $\theta = 0$ and $y(t) = z_4(t)$. Then the equation of the system can be written in the form

$$\ddot{x} + \Omega^2 x = -2a[1 + \xi(t)]\dot{x} + f(\dot{x}) + \zeta(t), \tag{1}$$

$$f(\dot{x}) = \begin{cases} k\dot{x} & (|\dot{x}| < d), \\ kd & (\dot{x} \geq d), \end{cases} \qquad f(-\dot{x}) = -f(\dot{x}),$$

where α, Ω, k, d are constants; $\xi(t), \zeta(t)$ are stationary random processes with correlation functions $K_\xi(\tau), K_\zeta(\tau)$; we assume these processes to be uncorrelated.

We shall regard the system as "quasi-harmonic" ; in the present note we shall also restrict ourselves to the case of small relative changes in the amplitude of self-oscillations. The latter assumption makes it possible to linearize the period-averaged equations in a neighborhood of the limiting cycle and then to apply correlation theory ([3],[6]). Introduce the notation

$$\alpha = \varepsilon\alpha_1, \qquad k = \varepsilon k_1, \qquad \xi(t) = \varepsilon\xi_1(t), \qquad \zeta(t) = \varepsilon^2\zeta_1(t) \tag{2}$$

and, passing in (1) to the new variables $b(t), \varphi(t)$,

$$x = b\sin(\Omega t + \varphi), \qquad \dot{x} = \Omega b\cos(\Omega t + \varphi), \tag{3}$$

we obtain a system of equations in the standard form

$$\dot{b} = \varepsilon X_1(b, \varphi, t) + \varepsilon^2 Y_1(b, \varphi, t), \qquad \dot{\varphi} = \varepsilon X_2(b, \varphi, t) + \varepsilon^2 Y_2(b, \varphi, t), \tag{4}$$

where the expressions Y_1, Y_2 contain, and the expressions X_1, X_2 do not contain, the random functions $\xi(t), \zeta(t)$. The first approximation for the mean amplitude of the self-oscillations b_0 is found from the condition

$$\int_0^{2\pi/\Omega} X_1(b_0, \varphi, t)\, dt = 0,$$

which, on the basis of (1)—(4), we reduce to the form

$$\alpha = (k/\pi)\{\arcsin(d/\Omega b_0) + (d/\Omega b_0)[1 - (d/\Omega b_0)^2]^{1/2}\}. \tag{5}$$

Next we linearize the problem, putting in (4)

$$b = b_0 + \varepsilon b_1 + \dots, \qquad \varphi = \varphi_0 + \varepsilon\varphi_1 + \dots$$

The random function $b_1(t)$ satisfies the linear equation

$$\dot{b}_1 + \beta b_1 = \varepsilon^{-1}\left[-2\alpha b_0 \xi(t)\cos^2(\Omega t + \varphi_0) + \Omega^{-1}\zeta(t)\cos(\Omega t + \varphi_0)\right], \tag{6}$$

$$\beta = 2[\alpha - (k/\pi)\arcsin(d/\Omega b_0)].$$

Writing the particular solution of equation (6) in the form of an integral with lower limit $-\infty$, we finally obtain (in the first approximation) the following relation for the known correlation function $R(\tau) = \varepsilon^2\langle b_1(t) b_1(t+\tau)\rangle$ of the random changes of amplitude:

$$R(\tau) = \frac{1}{2\beta}\int_{-\infty}^{\infty} H(\nu) e^{-\beta|\nu-\tau|}\, d\nu, \tag{7}$$

$$H(\nu) = \alpha^2 b_0^2 K_\xi(\nu)\left(1 + \frac{1}{2}\cos 2\Omega\nu\right) + \left(\frac{1}{2}\Omega^{-2}\right) K_\zeta(\nu)\cos\Omega\nu. \tag{8}$$

The solution of the integral equation of the first kind (7) has the form ([8])

$$H(\nu) = \int_{-\infty}^{\infty} \Phi(\omega)(\omega^2 + \beta^2) e^{-i\omega\nu}\, d\omega, \tag{9}$$

where $\Phi(\omega)$ is the spectral density of the process $\varepsilon b_1(t)$,

$$\Phi(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\tau) e^{i\omega\tau} \, d\tau. \tag{10}$$

It follows from (8) that, by varying the quantity l_0^2 (by changing the gain coefficient k in the feedback circuit), one can readily find the function $K_\xi(\nu)$ (in practice this solution will be sufficiently accurate if $\Phi(\omega)$ decreases sufficiently rapidly in the frequency range below Ω).

We note that the solution obtained for $K_\xi(\nu)$ is also valid in the case where the natural frequency of the system undergoes sufficiently slow random variations, since these variations will be "tracked" by the frequency of the self-oscillations (this is precisely the advantage, in the case under consideration, of the self-oscillatory identification scheme over the scheme considered in (4), with a sinusoidal signal applied to the input). In an analogous way, a solution can also be obtained in the case where, in order to describe the object, it is necessary to use a system of uncoupled equations of type (1) with respect to the generalized coordinates $x_j(t)$ (provided that for all l, r, s the condition $\alpha_l \ll |\Omega_r - \Omega_s|$ is satisfied). In this case each of the equations is investigated in turn, and for this purpose band-pass filters tuned to the frequencies Ω_r are inserted in LTs_1; the values Δ_l of the passband widths of the filters must satisfy the conditions $\alpha_l \ll \Delta_l \ll |\Omega_l - \Omega_{l-1}|, \; |\Omega_l - \Omega_{l+1}|$.

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Note: Figure translations are in progress. See original paper for figures.