ON THE NONLINEAR THEORY OF CURRENTS AT THE EQUATOR

GEOPHYSICS

1967

SovietRxiv

Abstract

Full Text

UDC 551.465

GEOPHYSICS

E. N. MIKHAILOVA, A. I. FEL' ZENBAUM, N. B. SHAPIRO

ON THE NONLINEAR THEORY OF CURRENTS AT THE EQUATOR

(Presented by Academician L. I. Sedov on 23 IX 1966)

The main features of the established wind-driven oceanic circulation, including the western intensification of currents and the existence of deep countercurrents in the equatorial region, have been obtained within the framework of the linear theory for a homogeneous fluid ([1]). The influence of the nonlinear terms in the equations of horizontal motion is manifested only near the western coast of the ocean and at the equator. In studying the effect of nonlinearity at the equator, it is apparently unnecessary to consider the entire ocean; it is sufficient to restrict oneself only to the region adjacent to the equator, on whose boundary, as in the case of the linear theory, the integral stream function ψ may be taken equal to zero. Taking into account that this region is elongated in the zonal direction and assuming the wind to be unchanging in this direction, we replace it by a closed zonal channel ([2]). We write the equations of motion, the boundary conditions, and the condition of closedness of the channel in the form

$$A\,\partial^2 u/\partial z^2 + \Omega v = -g\,\partial\zeta/\partial x + v\,\partial u/\partial y + w\,\partial u/\partial z - \Omega' w, \tag{1}$$

$$A\,\partial^2 v/\partial z^2 - \Omega u = -g\,\partial\zeta/\partial y + v\,\partial v/\partial y + w\,\partial v/\partial z;$$

$$\partial v/\partial y + \partial w/\partial z = 0; \tag{2}$$

$$\text{for } z = 0 \qquad \rho A\,\partial u/\partial z = -T_x, \qquad \rho A\,\partial v/\partial z = -T_y; \tag{3}$$

$$w = 0; \tag{4}$$

$$\text{for } z = H \qquad u = v = w = 0; \tag{5}$$

$$\text{for } y = \pm b \qquad S_y = 0; \tag{6}$$

$$\int_{-b}^{b} S_x \, dy = 0, \tag{7}$$

where $2b$ is the width of the channel, whose axis coincides with the equator; $\Omega' = 2\omega \cos\varphi$ is the second Coriolis parameter. For the remaining notation see ([3]).

In the case of a closed basin of arbitrary shape the problem reduces to the solution at each step of the elliptic equation

$$\vartheta' \Delta\psi + (\partial\vartheta'/\partial x + \partial\lambda'/\partial y)\,\partial\psi/\partial x + (\partial\vartheta'/\partial y - \partial\lambda'/\partial x)\,\partial\psi/\partial y =$$

$$= \operatorname{div}(n'T + \lambda' S^*) + \operatorname{rot}_z(m'T + \vartheta' S^*) \tag{8}$$

for the function ψ, which vanishes on the contour of the basin, and then to the subsequent computation of all the unknown functions by known formulas. First the linear problem is solved for $\Omega' = 0$ and $S_x^* = S_y^* = 0$, and u, v, w are found. Then S_x^*, S_y^* are computed, and equation (8) is solved again, and so on until the iterative process converges ([3]).

Let us apply the indicated method to solve the problem under consideration. For $T_x = T_x(y)$, $T_y = 0$, and $\psi = \psi(y)$, from equation (8) we obtain

$$S_x = \frac{1}{\vartheta'}\,(m'T_x + \vartheta' S_x^* - \lambda' S_y^* + c), \qquad S_y = 0, \tag{9}$$

where

$$c = \frac{1}{\displaystyle\int_{-b}^{b} \frac{1}{\vartheta'}\, dy} \int_{-b}^{b} \frac{1}{\vartheta'} \left(\lambda' S_y^* - \vartheta' S_x^* - m' T_x\right) dy. \tag{10}$$

As is seen from formula (16) of paper (3), the constant of integration $c = \partial\zeta/\partial x$.

Specific computations were carried out at the Computing Center of the Siberian Branch of the USSR Academy of Sciences. A grid with steps $\Delta y = b/10$ and $\Delta z = H/10$ was chosen. First the linear problem was solved with allowance for the second Coriolis parameter, and the numerical experiments showed that the solution can be obtained practically for any values of the parameters entering into it (H, b, T_x, A). Then the nonlinear problem was solved, and it turned out that the method converges directly only for sufficiently large values of the

Fig. 1

Figure 1: Fig. 1

coefficient of vertical exchange A, for example, for $A \geq 50$ cm^2/sec when $H = 200$ m, $b = 555$ km, $T_x = -0.2$ dyne/cm^2.

Fig. 1. Distribution of the zonal component of the current velocity at the equator (a) and of the zonal component of the total transport (b) in the case of a uniform wind for the following parameter values: 1—solution of the linear problem without allowance for $2\omega \cos\varphi \cdot w$ when $T_x = -0.2$ dyne/cm^2, $H = 200$ m, $A = 50$ cm^2/sec ($\Omega' = 0$); 2—solution of the linear problem with allowance for $2\omega \cos\varphi \cdot w$ for the same parameters ($\Omega' \neq 0$); 3—solution of the nonlinear problem for the same parameters; 4—solution of the nonlinear problem for $H = 200$ m, $T_x = -0.5$ dyne/cm^2, $A = 50$ cm^2/sec; 5—solution of the nonlinear problem for $H = 200$ m, $T_x = -0.2$ dyne/cm^2, $A = 30$ cm^2/sec.

However, the following special device makes it possible to extend the domain of convergence. First, a solution is sought for the maximum wind stress for which the method converges. For example, at $A = 30$ cm^2/sec we find a solution for $T_x \sim -0.05$ dyne/cm^2, for which the method converges. Then the obtained solution is used as the zero approximation for the case of a somewhat greater wind stress, and so on until we arrive at the required wind stress (in our example, at the value $T_x = -0.2$ dyne/cm^2). In this way, apparently, one can obtain a solution for practically all values of the entering parameters that are of interest to us; this is connected, however, with a large expenditure of machine time and with certain technical difficulties.

Let us consider the results of calculations obtained for a uniform wind ($T_x =$ const). In Figs. 1 and 2 these results are presented for the linear and nonlinear cases. In the linear case, without taking into account the component of the Coriolis force due to vertical motion ($\Omega' = 0$), the total transport at the equator is directed westward, but nevertheless there exists, at the equator itself, a subsurface countercurrent. It will exist for any values of T_x, H, b, A, since from formula (10), when $S_x^* = S_y^* = 0$, it always follows that $\partial\zeta/\partial x > -T_x/gH$.*

* The zonal component of the current velocity at the equator in the linear case is calculated by formula (2)

$$u = \frac{T_x(H-z)}{A} + \frac{g}{A}\,\frac{H^2 - z^2}{2}\,\frac{\partial\zeta}{\partial x}.$$

Taking into account the horizontal component of the Coriolis force caused by the vertical motion leads to a weakening of the undercurrent at the equator itself (curves 2); moreover, for sufficiently small values of the coefficient of vertical exchange ($A \leqslant 30$ cm^2/sec), when the role of the Coriolis force sharply increases, this undercurrent disappears altogether.

Figure 2

Figure 2: Figure 2

Let us note that the results obtained when the nonlinear inertial terms are taken into account, and when they are taken into account together with the component of the Coriolis force caused by the vertical motion, practically do not differ; for this reason below we shall speak only of the nonlinear problem. As is seen from Figs. 1 and 2, allowing for nonlinearity leads to a redistribution of the total flow with latitude in comparison with the linear case. Namely, nonlinearity leads to the creation at the equator and near it of a total flow directed eastward, which contributes to the strengthening of the undercurrent in this region. As is seen from a comparison of curves 3, 4, 5 in Fig. 1, the role of nonlinearity increases with increasing wind, and also with decreasing coefficient of vertical exchange. Allowing for nonlinearity leads to the equatorial undercurrent being drawn into a narrow jet with maximum velocities at the equator itself; moreover, in contrast to the linear case, this jet is separated from the remaining currents by a westward flow (Fig. 2).

Fig. 2. Isolines of the zonal component of the current velocity u in the case of a uniform wind.
a $-T_x = -0.2$ dyn/cm^2, $H = 200$ m, $A = 50$ cm^2/sec (linear problem);
b $-T_x = -0.2$ dyn/cm^2, $H = 200$ m, $A = 50$ cm^2/sec (nonlinear problem);
c $-T_x = -0.2$ dyn/cm^2, $H = 200$ m, $A = 30$ cm^2/sec (nonlinear problem)

The resulting picture of the distribution of the zonal component of the current velocity is in good agreement with observations. However, the obtained values of the velocity of the equatorial subsurface undercurrent prove to be somewhat underestimated, which is apparently due to the relatively large values of the coefficient of exchange of momentum at the equator. This conclusion is confirmed by the results of Charney ([4]) and Robinson ([5]), who considered the problem of currents at the equator itself for a prescribed zonal slope of the level $\partial\zeta/\partial x = -T_x/gH$. In the present work—and this is the main point—the slope of the level is determined from the solution of the problem and may vary depending on the parameters entering into it. For example, for $H = 200$ m, $b = 555$ km, $T_x = -0.2$ dyn/cm^2, $A = 50$ cm^2/sec, in the linear case it is equal to 1.2 cm per 1000 km, and in the nonlinear case to 1.16 cm per 1000 km. For $A = 30$ cm^2/sec, the zonal slope of the level in the linear case is 1.6 cm, and in the nonlinear case 1.11 cm per 1000 km.

Let us note that the patterns of the distribution of the meridional and vertical components of the current velocity in the examples considered remain practically the same as in the linear case. In the upper part the current

directed northward, and in the lower one southward, with the greatest velocities of both the northern and southern currents being attained near the equator (at the equator itself $v = 0$), contributing to the concentration of the equatorial

Fig. 3

Figure 3: Fig. 3

deep countercurrent at the equator, both in the linear and in the nonlinear cases. In a narrow zone near the equator, approximately up to 1° north and south of it, there is an ascent of waters ($w \sim -5 \cdot 10^{-3}$ cm/sec), while in the remaining region there is subsidence ($w \sim 5 \cdot 10^{-4}$ cm/sec). In all the examples considered, w does not change sign with depth.

Fig. 3. Curves of the distribution of wind stress and of the full flux with latitude in the linear (1) and nonlinear (2) cases for $H = 200$ m and $A = 50$ cm^2/sec

Figure 3 gives some results of calculations for the case of nonuniform wind. In the case where the linear theory already gives at the equator a full flux in the easterly direction—for example, when there exists a local calm zone near the equator—allowance for nonlinearity does not lead to an enhancement of this full flux, but even, on the contrary, to a certain weakening of it. Thus, nonlinearity is in some sense a stabilizing factor.

Marine Hydrophysical Institute
Academy of Sciences of the Ukrainian SSR

Received
20 IX 1966

REFERENCES

1. A. I. Fel' zenbaum, N. B. Shapiro, *DAN*, **168**, No. 3 (1966).
2. A. I. Fel' zenbaum, in the book *The Lomonosov Current*, 3, Kiev, 1966.
3. N. Mikhailova, A. I. Fel' zenbaum, N. B. Shapiro, *DAN*, **168**, No. 4 (1966).
4. J. Charney, *Deep-Sea Res.*, **6** (1960).
5. A. R. Robinson, *J. Marine Res.*, **24**, No. 2 (1966).

Note: Figure translations are in progress. See original paper for figures.