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ON THE RELATION BETWEEN STRESSES AND STRAINS IN NONLINEAR VISCOUS ELASTICITY

B. E. POBEDRYA

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Abstract

Full Text

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THEORY OF ELASTICITY

B. E. POBEDRYA

ON THE RELATION BETWEEN STRESSES AND STRAINS IN NONLINEAR VISCOUS ELASTICITY

(Presented by Academician L. I. Sedov, 18 VI 1966)

Let us fix some Lagrangian coordinate system ξ^k and consider the stress tensor $P^{ij}(\tau, \xi^k)$ and the strain tensor $E_{ij}(\tau, \xi^k)$ in the form of abstract functions $P(\tau)$ and $E(\tau)$, defined on the interval $[0, t]$ in the Hilbert spaces H_P and H_E, respectively.

Suppose that there exists a sufficiently smooth operator $P = F(E)$, mapping the space H_E into H_P, with $F(0) = 0$. For example, let all Fréchet derivatives at zero exist. Then one can write

$$P = \sum_{n=1}^{\infty} \Gamma_n E^n, \tag{1}$$

where

$$\Gamma_n E^n = \int_0^t \cdots \int_0^t \Gamma_n(t, \tau_1, \tau_2, \dots, \tau_n) E(\tau_1) \cdots E(\tau_n)\, d\tau_1 \cdots d\tau_n.$$

Denote by K_1 the linear operator inverse to Γ_1.

It can be shown that equations (1) are solvable:

$$E = \sum_{n=1}^{\infty} K_n P^n, \tag{2}$$

and, for fixed n, each kernel K_i $(i = 2, 3, \dots, n)$ is expressed explicitly in terms of the known kernels Γ_j $(j = 2, 3, \dots, i)$ and the kernel K_1. For example:

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$$K_2(t,\tau_1,\tau_2) = -\int_0^t K_1(t,\xi)\,d\xi \int_{\tau_1}^{\xi}\int_{\tau_2}^{\xi} \Gamma_2(\xi,\xi_1,\xi_2)K_1(\xi_1,\tau_1)K_1(\xi_2,\tau_2)\,d\xi_1 d\xi_2; \tag{3}$$

$$\begin{aligned} K_3(t,\tau_1,\tau_2,\tau_3) = & -\int_0^t K_1(t,\xi)\,d\xi \int_{\tau_1}^{\xi}\int_{\tau_2}^{\xi}\int_{\tau_3}^{\xi} \Gamma_3(\xi,\xi_1,\xi_2,\xi_3)K_1(\xi_1,\tau_1)\times \\ & \times K_1(\xi_2,\tau_2)K_1(\xi_3,\tau_3)\,d\xi_1 d\xi_2 d\xi_3 + 2\int_0^t K_1(t,\xi)\,d\xi \int_0^{\xi} d\xi_1 \int_{\tau_3}^{\xi} \Gamma_2(\xi,\xi_1,\xi_2)\times \\ & \times K_1(\xi_2,\tau_3)\,d\xi_2 \int_0^{\xi_1} K_1(\xi_1,\eta)\,d\eta \int_{\tau_1}^{\eta}\int_{\tau_2}^{\eta} \Gamma_2(\eta,\eta_1,\eta_2)K_1(\eta_1,\tau_1)K_1(\eta_2,\tau_2)\,d\eta_1 d\eta_2. \end{aligned} \tag{4}$$

And so on. Owing to this, the specification of the equations of state of the medium in the form (1) and (2) is completely equivalent.

Each kernel $\Gamma_n(t,\tau_1,\dots,\tau_n)$ is a tensor of rank $2(n+1)$ and, in the case of an isotropic medium, is expressed in the form of a sum of tensors, compos–

composed of products of the metric tensors δ_{ij} and certain scalars. In this case equations (4) can be written in the form

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$$
\begin{aligned}
E_{ij}(t) = \delta_{ij}\int_0^t K_{11}(t,\tau_1)P_k^k(\tau_1)\,d\tau_1 + \int_0^t K_{12}(t,\tau_1)P_{ij}(\tau_1)\,d\tau_1 + \\
+ \delta_{ij}\int_0^t\int_0^t K_{21}(t,\tau_1,\tau_2)P_k^k(\tau_1)P_l^l(\tau_2)\,d\tau_1 d\tau_2 + \delta_{ij}\int_0^t\int_0^t K_{22}(t,\tau_1,\tau_2)P^{kl}(\tau_1)\times \\
\times P_{lk}(\tau_2)\,d\tau_1 d\tau_2 + \int_0^t\int_0^t K_{23}(t,\tau_1,\tau_2)P_k^k(\tau_1)P_{ij}(\tau_2)\,d\tau_1 d\tau_2 + \\
+ \int_0^t\int_0^t K_{24}(t,\tau_1,\tau_2)P_i^k(\tau_1)P_{kj}(\tau_2)\,d\tau_1 d\tau_2 + \\
+ \delta_{ij}\int_0^t\int_0^t\int_0^t K_{31}(t,\tau_1,\tau_2,\tau_3)P_k^k(\tau_1)P_l^l(\tau_2)P_m^m(\tau_3)\,d\tau_1 d\tau_2 d\tau_3 + \\
+ \int_0^t\int_0^t\int_0^t K_{32}(t,\tau_1,\tau_2,\tau_3)P_k^k(\tau_1)P_m^l(\tau_2)P_{lm}(\tau_3)\,d\tau_1 d\tau_2 d\tau_3 + \\
+ \int_0^t\int_0^t\int_0^t K_{33}(t,\tau_1,\tau_2,\tau_3)P_k^k(\tau_1)P_l^l(\tau_2)P_{ij}(\tau_3)\,d\tau_1 d\tau_2 d\tau_3 + \\
+ \int_0^t\int_0^t\int_0^t K_{34}(t,\tau_1,\tau_2,\tau_3)P^{kl}(\tau_1)P_{lk}(\tau_2)P_{ij}(\tau_3)\,d\tau_1 d\tau_2 d\tau_3 + \\
+ \int_0^t\int_0^t\int_0^t K_{35}(t,\tau_1,\tau_2,\tau_3)P_k^k(\tau_1)P_i^l(\tau_2)P_{lj}(\tau_3)\,d\tau_1 d\tau_2 d\tau_3 + \\
+ \delta_{ij}\int_0^t\int_0^t\int_0^t K_{36}(t,\tau_1,\tau_2,\tau_3)P_l^k(\tau_1)P_m^l(\tau_2)P_k^m(\tau_3)\,d\tau_1 d\tau_2 d\tau_3 + \\
+ \int_0^t\int_0^t\int_0^t K_{37}(t,\tau_1,\tau_2,\tau_3)P_i^k(\tau_1)P_k^l(\tau_2)P_{lj}(\tau_3)\,d\tau_1 d\tau_2 d\tau_3 + \ldots
\end{aligned} \tag{5}
$$

In exactly the same form, equations (1) can be written, and the kernels of these equations are expressed through the kernels of equations (5) by formulas of the type (3) and (4).

If one requires that the equations of state (1) and (2) be invariant with respect to the choice of the time origin, then all the kernels become kernels of difference type.

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Note: Figure translations are in progress. See original paper for figures.

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