GAS-KINETIC COMPUTATION OF THE THERMAL GLIDING VELOCITY OF A GAS IN THE VICINITY OF A SOLID SURFACE

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Abstract

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Yu. I. YALAMOV, I. N. IVCHENKO,
Corresponding Member of the USSR Academy of Sciences B. V. DERYAGIN

GAS-KINETIC COMPUTATION OF THE THERMAL GLIDING VELOCITY OF A GAS IN THE VICINITY OF A SOLID SURFACE

In our previous works (^{1}, ^{2}) it was shown that, in order to determine the velocity of gas slip along a solid wall in the presence of a temperature gradient tangential to the wall, it is necessary first to calculate the velocity distribution function of the gas molecules with a more correct allowance than in work (^{1}) for the collision integral in the Boltzmann kinetic equation. The distribution function was determined by us in work (^{2}). In doing so we used numerical values of the integrals of the collision integral given in the work of American authors (^{3}). However, as shown by the analytical calculation carried out by us, the correct numerical values of these integrals differ substantially from the erroneous values given in work (^{3}). For this reason, in the present work we give the value of the distribution function corrected in comparison with work (^{2}), and on the basis of this new function we determine the velocity of thermal slip.

The analytical calculation carried out by us gives the following values of the integrals of the collision integral:

$$[c_y \operatorname{sign} c_x,\ c_y \operatorname{sign} c_x] = \frac{1 - 8\sqrt{2}}{12}\,\frac{\pi}{\lambda},$$

$$[c_y \operatorname{sign} c_x,\ c_x c_y] = -\frac{3\pi + 8}{32}\sqrt{\frac{2}{\pi}}\,\frac{\pi}{\lambda},$$

$$[c_x c_y,\ c_x c_y] = -\frac{2}{5}\,\frac{\pi}{\lambda},$$

$$[c_x c_y \operatorname{sign} c_x,\ c_x c_y \operatorname{sign} c_x] = \frac{-46 + 17\sqrt{2}}{120}\,\frac{\pi}{\lambda},$$

$$[c_y S_{3/2}^{(1)}(c^2),\ c_y S_{3/2}^{(1)}(c^2)] = -\frac{4}{3}\,\frac{\pi}{\lambda},$$

$$[c_y S^{(1)}_{3/2}(c^2)\,\mathrm{sign}\,c_x,\ c_y S^{(1)}_{3/2}(c^2)\,\mathrm{sign}\,c_x] = \frac{831 - 943\sqrt{2}}{192}\,\frac{\pi}{\lambda}.$$

The brackets are introduced to denote integrals of the form

$$[F(c), G(c)] = \int e^{-c^2} F(c)\, I(G)\, dc,$$

where F and G are functions of the corresponding components of the dimensionless velocity $c = (m/2kT)^{1/2} v$, and $I(G)$ is the linearized collision integral.

Let us consider a gas situated in the field of a temperature gradient tangential to an infinite wall. We choose the origin of coordinates on the surface of the wall. The x axis is directed along the normal, and the y axis along the surface. Along the y axis there is a temperature gradient in the gas.

The velocity distribution function of the molecules will have the form (^{2})

$$f = f^{(0)}\left[1 + 2c_{yu} + \tau c_y S^{(1)}_{3/2}(c^2)\frac{\partial \ln T}{\partial y} + \Phi(x, c)\right], \tag{1}$$

where $f^{(0)} = n(m/2\pi kT)^{3/2} e^{-mv^2/2kT}$; u is the dimensionless mass velocity of the gas, $\tau = {}^{15}/_{16}\lambda\pi^{1/2}$ (λ is the mean free path); $S^{(1)}_{3/2}(c^2) = {}^{5}/_{2} - c^2$.

The correction $\Phi(x, \mathbf{c})$ to the distribution function will be given by the expression

$$\begin{aligned}\Phi(x, \mathbf{c}) = {} & \frac{a_0^+ + a_0^-}{2} c_y + \frac{a_0^+ - a_0^-}{2} c_y\,\mathrm{sign}\,c_x + \frac{a_1^+ + a_1^-}{2} c_{xc} y \\ & + \frac{a_1^+ - a_1^-}{2} c_{xc} y\,\mathrm{sign}\,c_x + \frac{a_2^+ + a_2^-}{2} c_{xc} y + \frac{a_2^+ - a_2^-}{2} c_{xc} y\,\mathrm{sign}\,c_x \\ & + \frac{a_3^+ + a_3^-}{2} c_y S^{(1)}_{3/2}(c^2) + \frac{a_3^+ - a_3^-}{2} c_y S^{(1)}_{3/2}(c^2)\,\mathrm{sign}\,c_x.\end{aligned} \tag{2}$$

For the functions $a_i^\pm$, in the work (^{2}) systems of moment equations were obtained; their solution, taking into account the corrected values of the integrals from the collision integral, will have the form:

$$\begin{aligned} a_0^+ &= \alpha_0^+ c_1 e^{-2.201x/\lambda}, & a_0^- &= c_1 e^{-2.201x/\lambda}, \\ a_1^+ &= \alpha_1^+ c_1 e^{-2.201x/\lambda}, & a_1^- &= \alpha_1^- c_1 e^{-2.201x/\lambda}, \\ a_2^+ &= \alpha_2^+ c_2 e^{-0.5404x/\lambda}, & a_2^- &= c_2 e^{-0.5404x/\lambda}, \\ a_3^+ &= \alpha_3^+ c_3 e^{-1.150x/\lambda}, & a_3^- &= c_3 e^{-1.150x/\lambda}, \end{aligned} \tag{3}$$

where $\alpha_0^+ = 4.193,\ \alpha_1^+ = -4.127,\ \alpha_1^- = 0.5222,\ \alpha_2^+ = -5.101,\ \alpha_3^+ = 5.985.$

At large distances from the wall, all functions $a_i^{\pm}(x)$ vanish, and the distribution function will coincide with the Chapman–Enskog distribution (^{4}).

The constants c_i in expressions (3) are determined from the boundary conditions at the wall, which for the complete distribution function have the form

$$f^{+}(0, \mathbf{c}) = q f^{(0)} + (1-q) f^{-}(0, -c_x, c_y, c_z), \tag{4}$$

where

$$\begin{aligned} f^{+} &= f(x, y, c_x, c_y, c_z) \qquad \text{for } c_x > 0, \\ f^{-} &= f(x, y, c_x, c_y, c_z) \qquad \text{for } c_x < 0, \end{aligned}$$

q is the coefficient of diffuse reflection of molecules from the wall.

Substituting distribution function (1) into (4), taking (2) and (3) into account, we obtain

$$c_1 = -q \, \frac{2u}{\alpha_0^{+} - 1 + q},$$

$$c_2 = -q \, \frac{2u(\alpha_1^{+} - \alpha_1^{-} + q\alpha_1^{-})}{(\alpha_0^{+} - 1 + q)(1 - q - \alpha_2^{+})},$$

$$c_3 = -q\tau \frac{\partial \ln T}{\partial y} \, \frac{1}{\alpha_3^{+} - 1 + q}.$$

The presence of a solid wall does not change the x-component of the flux of the tangential component of momentum, since the combination entering this flux, $uc_{xc}y$, does not depend on temperature. In view of the fact that the wall does not change the x-component of the flux of the tangential component of momentum, the x-components of the flux of tangential momentum at the wall and at large distances from the wall must be equal. This physical condition can be written in the form

$$\int_{-} c_{xc} y f^{-}(0) \, d\mathbf{c} + \int_{+} c_{xc} y f^{+}(0) \, d\mathbf{c} = \int c_{xc} y f(\infty, \mathbf{c}) \, d\mathbf{c}, \tag{5}$$

where

$$\int_{-} d\mathbf{c} = \int_{c_x < 0} d\mathbf{c}, \qquad \int_{+} d\mathbf{c} = \int_{c_x > 0} d\mathbf{c}.$$

For the distribution function (1), condition (5) will have the form

$$\int_{+} c_x c_y e^{-c^2}\, dc \left\{[a_0^+(0) - a_0^-(0)]c_y + [a_1^+(0) + a_1^-(0)]c_{xc} y + \right.$$

$$\left. + [a_2^+(0) + a_2^-(0)]c_{xc} y + [a_3^+(0) - a_3^-(0)]c_y S^{(1)}_{3/2}(c^2)\right\} = 0. \tag{6}$$

Substituting the functions $a_i^{\pm}(0)$ into relation (6) and taking into account the values of the constants c_i, after simple calculations we obtain the expression for the thermal slip velocity V_{sk}:

$$V_{\text{sk}} = \frac{3}{2}\nu \frac{(a_3^+ - 1)(a_0^+ - 1 + q)(1 - q - a_2^+)}{(a_3^+ - 1 + q)\left[2(a_0^+ - 1)(1 - q - a_2^+) + (2 - q)\sqrt{\pi}\,(a_1^+ - a_1^- a_2^+)\right]} \frac{\partial \ln T}{\partial y},$$

where $\nu = {}^5/_{16}\lambda(2kT\pi/m)^{1/2}$ is the kinematic viscosity of the gas.

In the case of specular and diffuse reflection of gas molecules from the wall, the slip velocity will be given by the expressions:

for $q = 0$

$$V_{\text{sk}} = 0.8652\,\nu\,\partial \ln T/\partial y;$$

for $q = 1$

$$V_{\text{sk}} = 0.8913\,\nu\,\partial \ln T/\partial y.$$

For $q = 1$, the slip velocity given by our formula is 19% greater than the velocity according to Maxwell' s well-known formula (^{5}).

Institute of Physical Chemistry
Academy of Sciences of the USSR

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Note: Figure translations are in progress. See original paper for figures.

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