ON THE POSSIBILITY OF UNIDIRECTIONAL GENERATION IN A GAS TRAVELING-WAVE LASER

1967

SovietRxiv

Abstract

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UDC 539.186.22

PHYSICS

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ON THE POSSIBILITY OF UNIDIRECTIONAL GENERATION IN A GAS TRAVELING-WAVE LASER

(Presented by Academician A. M. Prokhorov, February 3, 1967)

The interaction of two waves traveling in opposite directions in a gas traveling-wave generator (TWG) was investigated in ([1,2]), and it was shown that if the frequencies ω_1, ω_2 of the waves are symmetric with respect to the line center, unidirectional generation (u.g.) occurs–one of the waves dies out. In the present work the possibility of u.g. is investigated when the number of generating modes is increased. We shall restrict ourselves to considering the interaction of four waves generating at two frequencies; at each frequency two waves may be generated, propagating in opposite directions.

The electromagnetic field in a TWG is the sum of traveling waves ([1,2])

$$E(z,t) = \sum_n E_n(t) e^{-i[\omega_n t + \varphi_n(t) - k_n z]} + \text{c.c.},$$

where $k_n = 2\pi q_n / L$, L is the resonator length, and q_n is a large integer. The amplitudes $E_n(t)$ satisfy the system of equations*

$$dE_n/dt = E_n(a_n - a_{nm} E_m^2) \qquad (n, m = 1, \dots, 4). \tag{1}$$

To determine the coefficients a_n, a_{nm}, the polarization of the medium was calculated, expressed through the off-diagonal elements of the density matrix of the atoms, averaged over the velocities v of thermal motion. The calculations were carried out by Lamb' s method ([3]); however, unlike Lamb, we did not use the approximation $c/L \approx |\omega_n - \omega_s| \gg v$.

One of the features of a gas laser is that the interaction of waves traveling in the same direction ($k_n \approx k_m,\ a_{nm} \equiv \chi_{nm},\ a_{nn} \equiv \beta$) differs from the interaction of waves traveling in opposite directions ($k_n \approx -k_m,\ a_{nm} \equiv \theta_{nm}$). Let us write the coefficients $\alpha, \beta, \theta, \chi$ for the case of an infinitely broad Doppler contour $ku : \eta \equiv \gamma_{ab}/ku \ll 1$ ($\gamma_{ab} \gg \gamma$–the linewidth of spontaneous emission of an individual atom, $\gamma \approx \gamma_a \approx \gamma_b$–the reciprocal lifetime of the lasing levels a, b).

$$\alpha_n = \frac{\delta\omega_p}{2}\left[\frac{N_0}{N_{\text{th}}}(1-\eta^2 f^2)-1\right]; \qquad \beta_n \equiv \frac{B\gamma_{ab}}{\gamma}\left[1-\eta^2(1+f_n^2)\right];$$

$$\theta_{nm} = B\left[\frac{\gamma_{ab}}{\gamma}\frac{1-\eta^2\left(1+\frac{1}{2}(f_n^2+f_m^2)\right)}{1+\frac{1}{4}(f_n+f_m)^2}+\eta^2\right];$$

$$\chi_{nm} = -\frac{B\gamma_{ab}}{\gamma\left(1+\left(\frac{f_n-f_s}{2}\right)^2\right)}\left[1+\frac{\gamma/\gamma_{ab}-\frac{1}{2}(f_n-f_s)^2}{\frac{\gamma}{\gamma_{ab}}\left[1+\left(\frac{\gamma_{ab}}{\gamma}(f_n-f_s)\right)^2\right]}\right]+O(\eta^2), \quad (2)$$

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* Equations (1) were obtained under the following approximations: 1) weak field $\alpha_n \ll \delta\omega_p/2$; 2) long field-relaxation time $T_{\text{rel}} \approx \alpha^{-1}$ compared with the lifetime γ^{-1}: $\alpha \ll \gamma$; 3) upon generation of no fewer than two waves in opposite directions there is one further approximation $\alpha \ll |\omega_n - \omega_s| = c/L$.

where

$$f_n = (\omega_n - \omega_0)/\gamma_{ab}; \qquad N_{\text{th}} = \hbar k u\, \delta\omega_p/4\pi^{3/2} d^2\omega_n;$$

$$B = \delta\omega_p N_0 d^2/N_{\text{th}}\hbar^2\gamma_{ab}^2;$$

$\delta\omega_p$ is the resonator line width; d is the nondiagonal matrix element of the dipole transition.

Let us consider a placement of the lasing frequencies $\omega_1 - \omega_c = \omega_0 - \omega_3$ symmetric with respect to the center of the line ω_0 (see Fig. 1). In this case the waves traveling in opposite directions at symmetric frequencies (1, 4 and 2, 3) interact strongly ($\theta_{23} > \beta$). As a consequence, the regime of simultaneous generation of such two waves is unstable [1]. It can be shown that all three-wave and four-wave lasing regimes are likewise unstable. Of the four waves, only two will lase. In this case two possibilities are possible: 1) two waves lase in opposite directions at one frequency (1 and 2 or 3 and 4); 2) two waves lase in one direction at different frequencies (1 and 3 or 2 and 4).

Fig. 1. Symmetric arrangement of two lasing frequencies. Dips in the gain contour

Let us consider the stability of both regimes. In the case of lasing at one frequency in different directions (1 and 2), the stationary solution has the form

Fig. 1. Symmetric arrangement of two lasing frequencies. Dips in the gain contour

Figure 1: Fig. 1. Symmetric arrangement of two lasing frequencies. Dips in the gain contour

$$E_1^2 = E_2^2 = \alpha/(\beta + \theta_{12}).$$

The stability condition with respect to the onset of waves at the frequency ω_3 is

$$\frac{1}{\delta E_3}\frac{d\delta E_3}{dt} \simeq \alpha - (\theta_{32} + \chi_{31})E_1^2 \simeq \frac{\alpha}{\beta + \theta_{12}}(\theta_{12} - \chi_{13}) < 0 \tag{3}$$

(here, neglecting terms of order η^2, we have put $\theta_{32} \simeq \beta$).

It is easy to show that the unidirectional-generation (u.g.) regime (1–3 or 2–4) is stable when the condition alternative to (3) is satisfied,

$$\frac{1}{\delta E_2}\frac{d\delta E_2}{dt} \simeq E_1^2(\chi_{13} - \theta_{12})\left(E_1^2 = E_3^2 = \frac{\alpha}{\beta + \chi_{13}}\right). \tag{4}$$

Substituting the values of θ_{12}, χ_{13} for $\eta = 0$ (see (2)), we find that the u.g. condition (4) is fulfilled for

$$\Delta\omega = |\omega_1 - \omega_3| > \sqrt{2\gamma\gamma_{ab}}. \tag{5}$$

The stability region $\delta\omega \equiv \frac{1}{2}(\omega_1 + \omega_3) - \omega_0$ (the deviation from the symmetric placement of the frequencies of the u.g. regime with respect to the onset of waves in opposite directions) is determined by the quantity $\delta_0\omega$: $\delta\omega \leq \delta_0\omega$

$$\left(\frac{\delta_0\omega}{\gamma_{ab}}\right)^2 = \frac{(\Delta\omega/\gamma_{ab})^2 - 2\gamma/\gamma_{ab}}{(\Delta\omega/\gamma_{ab})^4\gamma_{ab}/2\gamma + (\Delta\omega/\gamma_{ab})^2[\gamma/2\gamma_{ab} + 2\gamma_{ab}/\gamma - 1] + 4\gamma/\gamma_{ab}}. \tag{6}$$

The maximum value of $\delta_0\omega$ at $\gamma_{ab} = \gamma$ is of order $\delta_0\omega = 0.35\gamma$ and is attained at $\Delta\omega = 2.2\sqrt{\gamma\gamma_{ab}}$.

Thus, lasing regimes in resonators with frequency intervals $\Delta\omega = c/L > \sqrt{2\gamma\gamma_{ab}}$ and $c/L < \sqrt{2\gamma\gamma_{ab}}$ are qualitatively different. This difference is associated with the difference in the magnitude of the population modulation of a gas ensemble in the case of waves traveling in one direction and in the case of waves traveling in opposite directions. Indeed, in the generation of two waves in a TWG, the population difference of a group of atoms with

with any velocity v is modulated in space and time:

$$N(z,t,v) = N_0 \left\{ 1 - \frac{4d^2}{\hbar^2 \gamma_{ab} \gamma} \left[\sum_{n=1}^{2} \frac{E_n^2}{1 + [(\omega_n' - \omega_0)/\gamma_{ab}]^2} + \right. \right.$$
$$\left. \left. + \frac{2E_1 E_2 D_{12} \cos(\varphi_{12}(t,z) + \psi_{12})}{\sqrt{\{1 + [(\omega_1' - \omega_0)/\gamma_{ab}]^2\}\{1 + [(\omega_2' - \omega_0)/\gamma_{ab}]^2\}}} \right] \right\}. \quad (7)$$

$$D_{12} = \sqrt{\{1 + [(\omega_1' - \omega_0)/2\gamma_{ab}]^2\}\{1 + [(\omega_1' - \omega_2')/\gamma]^2\}^{-1}},$$

where $\varphi_{12}(t,z) = (\omega_1 - \omega_2)t + \varphi_1(t) - \varphi_2(t) - (k_1 - k_2)z$ is the phase difference of waves 1, 2; $\omega_n' = \omega_n - k_{nv}$ is the frequency of wave n in the frame of the moving atom. Attention is drawn to the fact that the phase $\varphi_{12}(t,z) + \psi_{12}$ of the population modulation is shifted relative to the phase $\varphi_{12}(t,z)$ of the modulation of the field energy

$$|E(z,t)|^2 = E_1^2 + E_2^2 + 2E_1 E_2 \cos \varphi_{12}(t,z). \quad (8)$$

This shift ψ_{12} is determined by the wave frequencies in the frame of the moving atom, ω_1', ω_2'. If these frequencies are equal, the shift ψ_{12} is absent, $\psi_{12} = 0$. Let us write the expression for ψ_{12} in the case of a symmetric arrangement of frequencies $\omega_1' - \omega_0 = \omega_0 - \omega_2'$

$$\operatorname{tg} \psi_{12} = -(\omega_1' - \omega_2')(\gamma + 2\gamma_{ab})/[2\gamma\gamma_{ab} - (\omega_1' - \omega_2')^2]. \quad (9)$$

The difference of the wave frequencies in the atom's frame is $\omega_1' - \omega_2' = \omega_1 - \omega_2 - (k_1 - k_2)v$. If the waves propagate in different directions, then $k_1 \approx -k_2$, and the frequency difference $\omega_1' - \omega_2'$ depends on the atom's velocity: $-ku < \omega_1' - \omega_2' < ku$. In this case, since $ku \gg \gamma_{ab}$, the phase shift φ_{12} (see (9)) varies from $-\pi$ to π, and the total value of the population modulation of the gas ensemble is equal to zero, more precisely $o(\eta^2)$.

If, however, the waves propagate in the same direction, then $k_1 \approx k_2$, and the frequency difference $\omega_1' - \omega_2'$ does not depend on the atom velocities. Consequently, the phase shift of the population modulation is constant for all atoms. Therefore there exists a population modulation of the ensemble different from zero, and it is shifted in phase relative to the phase of the modulation of the field energy by the amount ψ_{12}. If the wave frequencies are close, $\omega_1 \approx \omega_2$, then $\psi_{12} \approx 0$ (see (9)), i.e. the population is modulated in antiphase with the modulating field energy (see (7), (8))—the population is maximal where the field is minimal. Therefore the forced emission of the laser in this case,

$$S = \int N|E|^2 \, dz,$$

is less than in the case of a population constant in space, which occurs in the generation of waves in opposite directions. As a result, the unidirectional-generation regime for $\Delta\omega < \sqrt{2\gamma\gamma_{ab}}$ is energetically unfavorable, and the single-frequency regime is realized. If, however, the frequency difference $\Delta\omega$ is large, $\Delta\omega > \sqrt{2\gamma\gamma_{ab}}$, then the phase shift $|\psi_{12}| > \pi/2$. In this case the maxima of the populations are located in the same places as the maxima of the field energy. As a result, the forced emission in the unidirectional-generation regime is greater than the forced emission for a constant population difference (i.e. in regime 1-2 or 3-4). Therefore, for a large frequency difference (5), the two-wave unidirectional-generation regime is energetically advantageous and, as a result, stable.

The author expresses gratitude to E. E. Fradkin and M. P. Chaika for useful discussions, and also to N. I. Kaliteevskii for attention to the work.

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Received
9 II 1967

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Note: Figure translations are in progress. See original paper for figures.