EQUATIONS OF MOTION OF A MACHINE UNIT WITH A VARIATOR

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Fig. 1

Figure 1: Fig. 1

Abstract

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EQUATIONS OF MOTION OF A MACHINE UNIT WITH A VARIATOR

The present work is devoted to the consideration of the dynamic properties of the mechanical system engine–variator–working machine (EVW). As a model of the EVW we adopt the model shown in Fig. 1.

The driving shaft I and the driven shaft II have rotational motion with angular velocities ω and Ω, respectively. The driving torque M_{d} of engine D is assumed to depend on the speed, i.e. $M_{\mathrm{d}} = M_{\mathrm{d}}(\omega)$. A useful resistance torque M_{c} is applied to the working machine P. The variator B provides a transmission ratio i equal to

$$i = \omega/\Omega. \tag{1}$$

We shall regard the transmission ratio as a known independent function of time, $i = f(t)$. This is valid under automatic control from a separate engine. When the variator is controlled from the main engine, the transmission ratio may be regarded as an independent function of time if the power expended on control is considerably less than the power of the main engine.

As the variator we take a generalized nonimpulsive ideal variator, i.e. a variator without slip and with efficiency $\eta = 1$. Friction variators are closest to ideal variators; in them, by various design measures, geometrical slip is reduced to a minimum. The presence of a variator in a mechanical system imposes certain features on the construction of the calculation scheme. Thus, when reducing the real system to an idealized one (Fig. 1), it is necessary to reduce all forces and masses to two shafts–the driving and driven shafts of the variator.

Fig. 1

In this work only systems with constant reduced moments of inertia are considered, i.e. J_1–the moment of inertia of all links from the engine up to and including the driving shaft of the variator, reduced to shaft I; J_2–the moment of inertia of all links from the driven shaft of the variator to the working member, reduced to shaft II—are assumed constant, i.e. $J_1 = \text{const}$, $J_2 = \text{const}$.

After all forces and masses have been reduced to the two shafts, it is necessary to set up the differential equations of motion; in doing so, the prescribed law of variation of the transmission ratio, written in the form

$$\omega - i(t)\Omega = 0, \tag{2}$$

must be regarded as a constraint equation.

The principal feature of mechanical systems with variators is that this constraint is not of the holonomic type, and in deriving the equations of motion one must use the Appell or Chaplygin equations, or the first form of Lagrange' s equations with undetermined multipliers [1].

Let us apply Appell' s equations to the calculation of our system:

$$\sum_{i=1}^{n} \frac{\partial S}{\partial \ddot{\varphi}_i} = Q_i. \tag{3}$$

Here S is the energy of accelerations,

$$S = \sum_{i=1}^{l} \frac{J_i \ddot{\varphi}_i}{2}, \tag{4}$$

$\ddot{\varphi}$ is the second derivative of the generalized coordinate (in the present case, the angle of rotation) with respect to time; Q is the generalized force.

The energy of accelerations (4) for our case has the form

$$S = \frac{1}{2} J_1 \ddot{\varphi}^2 + \frac{1}{2} J_2 \ddot{\Phi}^2. \tag{5}$$

As is known [2], one of the coordinates, taken as dependent, must be eliminated from equation (5) before differentiating according to equation (3). For this one may use the constraint equation (2), differentiating it with respect to time.

We shall take as the dependent coordinate φ, the angle of rotation of the driving shaft I, and eliminate this coordinate from equation (5). Then the energy of accelerations takes the form

$$S = \left(\frac{J_1}{2} i^2 + \frac{J_2}{2}\right) \left(\frac{d\Omega}{dt}\right)^2 + J_1 i \frac{di}{dt} \Omega \frac{d\Omega}{dt} + \frac{J_1}{2} \left(\frac{di}{dt}\right)^2 \Omega^2. \tag{6}$$

The generalized force in equation (3) is determined, as usual, from the equation of elementary works,

$$M_\mathrm{d}\, d\varphi - M_\mathrm{c}\, d\Phi = 0. \tag{7}$$

From equation (7) it is also necessary to eliminate the dependent coordinate φ by means of the constraint equation.

After carrying out all operations in Appell' s equation (3), we obtain the differential equation of motion of the DVP system, written with respect to the angular velocity of the driven shaft:

$$(J_1 i^2 + J_2)\,\frac{d\Omega}{dt} + J_1 i \frac{di}{dt}\Omega = iM_\mathrm{d} - M_\mathrm{c}. \tag{8}$$

In an entirely analogous way, taking as the dependent velocity the angular velocity Ω of the driven shaft II, one may write the differential equation of motion of the DVP system with respect to the angular velocity ω of the driving shaft I. We have

$$(J_1 + J_2 y^2)\,\frac{d\omega}{dt} + J_2 y \frac{dy}{dt}\omega = M_\mathrm{d} - yM_\mathrm{c}. \tag{9}$$

For a more compact notation in this case, it is more convenient, instead of the transmission ratio i, to introduce its reciprocal function y, i.e.,

$$y = 1/i = \Omega/\omega = f(t). \tag{10}$$

Equations (8) and (9) are, of course, equivalent, and the choice of one or the other depends on the conditions posed in the specific problem.

The writing of equations (8) and (9) can be simplified somewhat by introducing new relative variables and making certain transformations. Then equation (8) takes the form

$$(x + 1/A^2)\, d\widetilde{\Omega}/d\tau + 0.5\,\widetilde{\Omega}\, dx/d\tau = m_\mathrm{e}\sqrt{x} - 1. \tag{11}$$

Here

$$\begin{gathered} x = i^2/\mu^2; \qquad \mu = M_c/M_n; \qquad A = \mu/\xi; \qquad \xi = \sqrt{J_2/J_1}; \\ \Omega = \Omega/\Omega_n; \qquad \Omega_n = \omega_n/\mu; \qquad \tau = t/T_d; \qquad T_d = J_1\omega_n/M_n; \\ m_d = M_d/M_n, \end{gathered} \tag{12}$$

and ω_n and M_n are the nominal angular velocity and nominal torque of the motor.

When introducing relative variables according to condition (12), the equilibrium state of the system at nominal load and motor speed was taken as the basis.

Equation (9) can be simplified in an analogous way:

$$(1+z)\,d\tilde{\omega}/d\tau + 0.5\tilde{\omega}\,dz/d\tau = m_d - A\sqrt{z}. \tag{13}$$

Here $\tilde{\omega} = \omega/\omega_n$; $z = \xi^2 y^2$, and the remaining notation has the same meaning as in conditions (12).

Equations (8) and (9), as well as (11), (13), may have various degrees of complexity depending on the type and character of the changes in the functions characterizing the torques M_d and M_c.

Let us take, for example, the motor characteristic $M_d = M_d(\omega)$ to be linear, of the form

$$M_d = a - b\omega, \tag{14}$$

or, in relative coordinates,

$$m_d = \gamma + (1-\gamma)\tilde{\omega}, \tag{15}$$

where γ is the multiplicity of the motor starting torque.

We shall assume the resisting torque M_c to be constant, i.e.,

$$M_c = \text{const}. \tag{16}$$

Then, taking conditions (14) and (16) into account, equations (8) and (9) can be written in the form

$$(J_1 i^2 + J_2)\Omega' + (J_1 i i' + b i^2)\Omega = ai - M_c; \tag{17}$$

$$(J_1 + J_2 y^2)\omega' + (J_2 y y' + b)\omega = a - yM_c, \tag{18}$$

and equation (13) in the form

$$(1+z)\tilde{\omega}' + (\gamma - 1 + 0.5z')\tilde{\omega} - A\sqrt{z}. \tag{19}$$

Equations (17) and (18), as well as (19), are linear with respect to angular velocity; the variables i, y, and z are known functions of time. The solution of these equations can immediately be written in explicit form. For example, the solution of (17) has the form

$$\Omega = \left\{ \exp\left[-\int_0^t \frac{bi^2\,dt}{J_1 i^2 + J_2} \right] \Big/ \sqrt{J_1 i^2 + J_2} \right\} \times$$

$$\times \left\{ \Omega_0 + \int_0^t \left[(ai - M_c) \exp\left[\int_0^t \frac{bi^2 dt}{J_1 i^2 + J_2} \right] \Big/ \sqrt{J_1 i^2 + J_2} \right] dt \right\}. \qquad (20)$$

Despite the explicit form, it is not easy to draw a conclusion about the form of the function from (20), because in most cases the quadratures cannot be evaluated.

If, instead of a function of the form (14), the motor characteristic is approximated by another function, for example by a parabola, then equation (17) is not integrable in quadratures at all, and it is considerably more difficult to draw any conclusion in this case. To avoid these difficulties, the authors constructed a block diagram and solved the problem of analyzing the indicated system

DVR on an electronic analog computer. In addition, a special graphical method for solving this problem was developed ([3]). Thus, if the law of variation of the transmission ratio $i = f(t)$ is known, then it is always possible to determine the law of motion of the DVR system.

From the structure of equations (8) and (9) it is evident that the motion of the system is influenced not only by the magnitude of the transmission ratio, but also by the rate of its variation; moreover, this influence has a damping character: the more sharply the transmission ratio changes, the less "obediently" the velocity of motion of the system changes.

Thus, in most cases the problem of analyzing the DVR system considered can be solved by applying an analytical or graphical method, or by using analog electronic modeling devices.

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Note: Figure translations are in progress. See original paper for figures.