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Linear boundary value problems with small lag

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Abstract

It is proved that in linear boundary value problems, the property of a parameter being an eigenvalue or a regular value is preserved under small changes in the delay. Bibliography: 6 items.

Full Text

Introduction

In 1967, M. I. El' sgol' ts and V. R. Nosov [**?**] investigated systems of differential equations with deviating arguments. We consider a system of the form:

$$x'(t) = \sum_{j=1}^{2} a_{1j}(t)x_j(t - \tau_j(t))$$

$$y'(t) = \sum_{j=1}^{2} a_{2j}(t)x_j(t - \tau_j(t))$$

subject to the boundary conditions:

$$x(a)\sin\alpha - y(a)\cos\alpha = 0$$
$$x(b)\sin\beta - y(b)\cos\beta = 0$$

where the initial functions are defined for $t-\tau(t) < a$ as $x(t-\tau(t)) = x(a)x_0(t-\tau(t))$ and $y(t-\tau(t)) = y(a)y_0(t-\tau(t))$, with $x_0(a) = 1$ and $y_0(a) = 1$. This formulation follows the general framework established in [**?**] and [**?**]. We assume that for $t \in [a, b]$, the delays satisfy $\tau_i(t) \geq 0$ and the maximum delay is denoted by $\Delta = \max \tau_i(t)$.

To analyze the eigenvalues of the system (1)-(4), we introduce a parameter λ and consider the modified system:

$$x'(t) = a_{11}(t, \lambda)x(t) + a_{12}(t, \lambda)y(t)$$
$$y'(t) = a_{21}(t, \lambda)x(t) + a_{22}(t, \lambda)y(t)$$

where the coefficients $a_{kj}(t,\lambda)$ are continuous functions of their arguments. Let $z(t,\lambda) = \{x(t,\lambda), y(t,\lambda)\}$ be a solution satisfying the initial conditions $x(a) = \cos\alpha$ and $y(a) = \sin\alpha$. As shown in [**?**], the norm $|z(t,\lambda)|^2 = x^2(t,\lambda) + y^2(t,\lambda)$ is non-zero for all $t \in [a,b]$. Furthermore, the solutions satisfy the continuity estimates $|x(t,\lambda) - \tilde{x}(t,\lambda)| < C_1\Delta$ and $|y(t,\lambda) - \tilde{y}(t,\lambda)| < C_2\Delta$.

Phase Analysis and Eigenvalues

We introduce the polar coordinate transformation by defining the phase function $\phi(t,\lambda) = \arctan(y(t,\lambda)/x(t,\lambda))$. The derivative of the phase function is given by:

$$\phi'(t,\lambda) = \frac{x(t,\lambda)y'(t,\lambda) - x'(t,\lambda)y(t,\lambda)}{x^2(t,\lambda) + y^2(t,\lambda)}$$

Substituting the system equations, we obtain an expression for $\phi'(t,\lambda)$ that depends on the coefficients a_{kj} and the delayed phase $\phi(t-\tau_i(t))$. Specifically, the evolution of the phase is governed by:

$$\Phi'(t,\lambda) = a_{21}(t,\lambda)\cos^2\phi(t,\lambda) + [a_{22}(t,\lambda) - a_{11}(t,\lambda)]\cos\phi(t,\lambda)\sin\phi(t,\lambda) + a_{12}(t,\lambda)\sin^2\phi(t,\lambda)$$

with the initial condition $\phi(a,\lambda) = \alpha$. The boundary condition at $t = b$ implies that for an eigenvalue λ_k, the phase must satisfy $\phi(b,\lambda_k) = \beta + k\pi$.

By applying the comparison theorems for differential equations [**?**, **?**], we can establish bounds on the phase function. If the coefficients satisfy certain monotonicity conditions with respect to λ, specifically that $\phi(b,\lambda)$ is a strictly increasing function, then there exists a unique sequence of eigenvalues λ_k corresponding to each integer k. The difference between the eigenvalues of the original system and the perturbed system can be bounded by a constant proportional to the maximum delay Δ.

Conclusion

The analysis demonstrates that the qualitative behavior of the Sturm-Liouville problem for differential equations with deviating arguments mirrors that of classical ODEs, provided the delays $\tau_i(t)$ are sufficiently small. The existence and distribution of eigenvalues are determined by the rotation of the solution vector in the phase plane. These results extend the findings of A. B. Nersesyan [**?**] and provide a basis for numerical methods in solving boundary value problems with delay.

References

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