NON-EXPONENTIAL DECAY LAW AND INTERFERENCE EXPERIMENTS

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Abstract

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NON-EXPONENTIAL DECAY LAW AND INTERFERENCE EXPERIMENTS

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1. As is known (1), the decay probability $L(t)$ is defined as $L(t) = |p(t)|^2 = M^2(t)$, where

$$p(t) = \int_{E_{\min}}^{\infty} e^{-\frac{i}{\hbar}Et}\,\omega(E)\,dE = M(t)\exp[iN(t)]. \tag{1}$$

Here $\omega(E)$ is the energy distribution (mass distribution) of the decaying state; $E_{\min}$ is the lower bound of the energy distribution, determined by the rest masses of the decay products.

It was shown (2) that, by virtue of the spectrality principle ($E_{\min} \geq 0$), $L(t)$, for no $\omega(E)$, can decrease exponentially as $t \to \infty$, since

$$\int_{-\infty}^{\infty} \frac{|\log M(t)|}{1+t^2}\,dt < \infty. \tag{2}$$

Deviations from the exponential decay law are determined, generally speaking, by the fine details of the behavior of $\omega(E)$ (2−4). In the usual pole assumption concerning the character of $\omega(E)$,

$$\omega(E) = \frac{1}{\pi}\,\frac{\Gamma}{(E-E_0)^2+\Gamma} \tag{3}$$

for $t \gg \hbar/E_0$*, i.e., at times much greater than the "period of intrinsic oscillations" (for elementary particles $\hbar/E_0 \sim 10^{-24}$ sec), the asymptotic formula (2,3) is valid

$$p(t) \simeq \exp\left[-\frac{i}{\hbar}E_0 t - \frac{\Gamma}{\pi}t\right] - \frac{i}{\pi}\,\frac{\Gamma\hbar}{(E_0^2+\Gamma^2)t}, \tag{4}$$

and, consequently,

$$N(t)\,\mathrm{arc\,tg}\left\{-\,\mathrm{tg}\,\frac{E_0 t}{\hbar}-\frac{1}{\pi}\frac{\Gamma\hbar e^{\Gamma t/\hbar}}{(E_0^2+\Gamma^2)t}\,\mathrm{cosec}\,\frac{E_0 t}{\hbar}\right\}. \tag{4a}$$

It follows from (4) that the contribution of the non-exponential term to the decay probability $L(t)$ is large in comparison with the contribution from the main exponential term only if

$$\frac{\Gamma t}{\hbar}e^{-\Gamma t/\hbar} \ll \frac{\Gamma}{E_0^2+\Gamma^2} \simeq \left(\frac{\Gamma}{E_0}\right)^2. \tag{5}$$

Since for ordinary unstable physical systems (particles, excluding resonances) $\Gamma/E_0 \ll 1$, (5) is satisfied at very large $t \sim 100\hbar/\Gamma$; consequently, direct experimental detection of the non-exponential term in the decay law is practically impos-

* For $\Gamma/E_0 \ll 1$ this coincides with the exact definition of the asymptotic region $t \gg \hbar/\sqrt{E_0^2+\Gamma^2}$ ([2,3]).

possible, since at such large t the statistical accuracy at real initial intensities is negligible.

At the same time, the presence of a nonexponential term is of fundamental interest for many physical problems ([4–8]), and observation of this term, which depends essentially on the details of the behavior of $\omega(E)$*, could, in particular, help solve an important problem: whether or not the properties of unstable particles depend on their preparation ([4–8]).

2. From this point of view, interference experiments (of the Pais–Piccioni effect type ([9,10])), which depend essentially not only on the modulus $M(t)$, but also on the phase (argument) $N(t)$ of the function $p(t)$, are of great interest. As follows directly from (4a), the contribution of the nonexponential term to the phase $N(t)$ is large in comparison with the contribution from the principal exponential term if

$$\frac{\Gamma\hbar}{(E_0^2+\Gamma^2)t} \gg e^{-\Gamma t/\hbar}\sin\frac{E_0 t}{\hbar}. \tag{6}$$

It follows immediately from (6) that this is certainly true in the vicinity of

$$t_n = \frac{\hbar}{E_0}n\pi,$$

where n is a natural number, i.e., the contribution of the nonexponential term to the phase $N(t)$ is large even at small t of order $\hbar/\Gamma$**, for which the contribution of this same nonexponential term to the decay probability $L(t)$, on the basis of (5), is negligibly small.

Let us estimate the region Δt around t_n where the contribution to the phase $N(t)$ from the nonexponential term is large. Assuming that Δt is small, we obtain

$$\Delta t \ll \frac{1}{\pi}\frac{\Gamma}{E_0}\frac{1}{n\pi}\, e^{\frac{\Gamma}{E_0}n\pi} \tag{7}$$

for $t_n \sim \hbar/\Gamma$; for example, this region is quite small,

$$\Delta t \sim \frac{1}{\pi}\left(\frac{\Gamma}{E_0}\right)^2.$$

3. Let us consider a typical interference experiment. Let the vector of the initial state ψ be a coherent superposition of states with vectors ψ_1 and ψ_2,

$$\psi = a\psi_1 + b\psi_2, \tag{8}$$

where a, b are certain constants; ψ_1 and ψ_2 evolve in time according to pole energy distributions***. Then

$$\begin{aligned}\psi(t) \simeq a\exp\left[-i\frac{E_1 t}{\hbar} - \frac{\Gamma_1}{\hbar}t\right] - \frac{ai}{\pi}\frac{\Gamma_1\hbar}{(E_1^2+\Gamma_1^2)t}\\ + b\exp\left[-i\frac{E_2 t}{\hbar} - \frac{\Gamma_2}{\hbar}t\right] - \frac{bi}{\pi}\frac{\Gamma_2\hbar}{(E_2^2+\Gamma_2^2)t}\end{aligned} \tag{9}$$

for $t \gg \max(\hbar/E_1, \hbar/E_2)$. Taking into account only the exponential terms in (9), we obtain

$$|\psi(t)|^2 = |a|^2 e^{-2\Gamma_1 t/\hbar} + |b|^2 e^{-2\Gamma_2 t/\hbar} + 2|a||b| e^{-(\Gamma_1+\Gamma_2)t/\hbar}\cos\left[\frac{(E_2-E_1)}{\hbar}t + \arg a - \arg b\right], \tag{10}$$

* At the same time, the principal exponential term in the decay law gives only information about Γ and does not even depend on E_0, let alone on the details of $\omega(E)$.

** At these small t, the statistical accuracy may be very high owing to the large initial intensity.

*** In addition to K^0, similar states can be obtained as follows: suppose there is a doubly degenerate atomic (or nuclear) level; by switching on a magnetic field, as a result of Zeeman splitting we precisely obtain the coherent superposition (8), and by changing the strength of the magnetic field one can make $\Delta E = E_2 - E_1$ arbitrary.

i.e., the typical periodic dependence on t, on the basis of which very small $\Delta E = E_2 - E_1$* can be determined. Thus, in the case $K^0 = \frac{1}{\sqrt{2}}(K_1^0 + K_2^0)$ (10), it was possible to determine $\Delta E = |m_{K_2^0} - m_{K_1^0}| \sim 0.5\,\frac{\hbar}{\Gamma_1}$**.

Taking account of the nonexponential terms (9) will plainly violate this periodic dependence of the interference term. With sufficiently large statistics (large initial intensity), one can in principle, from the deviation of the interference term from the periodicity predicted by (10),*** determine the contribution from the nonexponential terms. The contribution from the nonexponential terms in the interference term will be large in comparison with the contribution from the exponential terms if

$$\left|e^{-\Gamma_1 t/\hbar}\sin\frac{E_1 t}{\hbar} - e^{-\Gamma_2 t/\hbar}\sin\frac{E_2 t}{\hbar}\right| \ll \left|\frac{1}{\pi}\frac{\Gamma_1\hbar}{(E_1^2+\Gamma_1^2)t} - \frac{1}{\pi}\frac{\Gamma_2\hbar}{(E_2^2+\Gamma_2^2)t}\right|. \qquad (11)$$

It is clear that (11) will also be fulfilled for not very large $t \sim \min(\hbar/\Gamma_1, \hbar/\Gamma_2)$, i.e., for such t for which the statistical accuracy is large.

4. As was already indicated, the form of the nonexponential terms depends essentially on the detailed behavior of $\omega(E)$. Therefore the estimates obtained above for the contribution of the nonexponential term to the results of an interference experiment are, of course, purely indicative, since they are most essentially connected with the assumption of the pole character of the energy distribution (mass distribution), which is precisely what is of interest to test.

On the basis of investigations of the inverse problem in the quantum theory of decay ([2,3]), we obtain, in the general case, without any assumptions about the energy (mass) distribution $\omega(E)$, instead of (10),

$$|\psi(t)|^2 = |a|^2 M_1^2(t) + |b|^2 M_2^2(t) +$$

$$+2|a||b|M_1(t)M_2(t)\cos\left[\frac{2t}{\pi}P\int_0^\infty \frac{\log M_1(t') - \log M_2(t') - \log M_1(t) + \log M_2(t)}{{t'}^2 - t^2}\,dt'\right] +$$

$$+\sum_k\left[\operatorname{arc\,tg}\frac{2(t-\alpha_k^{(1)})\beta_k^{(1)}}{(t-\alpha_k^{(1)})^2 - (\beta_k^{(1)})^2} - \operatorname{arc\,tg}\frac{2(t-\alpha_k^{(2)})\beta_k^{(2)}}{(t-\alpha_k^{(2)})^2 - (\beta_k^{(2)})^2}\right], \qquad (12)$$

where $t_k^{(l)} = \alpha_k^{(l)} - i\beta_k^{(l)}$, $\beta_k^{(l)} > 0$ $(l = 1, 2)$, are the possible complex zeros of the functions $p_1(t)$ and $p_2(t)$, respectively. Thus, the interference experiment contains information about the complex zeros of $p(t)$, which, as shown in ([4]), may correspond to logarithmic singularities of $\omega(E)$. From (12) it is obvious that the complex zeros of $p(t)$ lead, generally speaking, to a deviation from periodicity in t of the interference term.

5. It is interesting to note that the nonexponential character of the decay law caused, according to the assumption of Goldberger and Watson ([11]), by the fact that the energy distribution $\omega(E)$ contains poles of order higher than the first, does not change, as can be seen directly, the argument of the cos in the interference term, i.e., nonexponentiality of this type precisely cannot be determined by an interference experiment, and therefore the results of determining the mass difference of K_1^0 and K_2^0 remain valid even if K_1^0 and K_2^0 were described by poles of higher order.

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* Interference experiments make it possible to determine precisely small ΔE; large ΔE, however, are difficult to determine in an interference experiment.

** This is certainly true in the study of the leptonic channel.

*** The deviation from periodicity can be determined with great statistical accuracy.

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Note: Figure translations are in progress. See original paper for figures.