SovietRxiv

Soviet-era science, translated into English

ESTIMATING THE COMPLEXITY OF MODELING ELECTRICAL LINES WITHOUT LOSSES

V. A. MALYSHEV

1967

SovietRxiv

View the original and related papers at https://sovietrxiv.org/items/ru-196701.66149

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

Abstract

Full Text

UDC 518.61

CYBERNETICS AND CONTROL THEORY

V. A. MALYSHEV

ESTIMATING THE COMPLEXITY OF MODELING ELECTRICAL LINES WITHOUT LOSSES

(Presented by Academician A. N. Kolmogorov, January 23, 1967)

Various modifications of the definition of the complexity of realizing discrete functions by cybernetic control systems are well known. However, the question of the complexity of certain cybernetic methods for solving practical problems does not fit into the developed schemes. Thus, in practice the method of electrical modeling is often used ([2], [4]). In the present note the problem of estimating the complexity of electrical modeling is formulated as the problem of approximation by a certain class of rational functions. Upper and lower estimates of the complexity are obtained.

1. Complexity of electrical circuits with lumped parameters. By a passive electrical circuit (A, φ) with lumped parameters we shall mean a connected nonseparable graph A together with a function φ that assigns to each pair of edges (b_i, b_j) of this graph a number L_{ij}, $i \neq j$, and to each edge b_i three numbers $L_i, C_i, R_i \geq 0$ ([1]). We shall speak of a circuit without losses if all R_i are equal to zero. The complexity $S(A, \varphi)$ of the circuit (A, φ) will be defined as the total number of numbers L_{ij}, L_i, C_i, R_i that are different from zero.

For any integer $n \geq 0$ introduce the class W_n of rational functions of the form:

$$Z(\omega) = k_\infty p + \frac{k_0}{p} + \sum_{i=1}^{n} \frac{k_i p}{p^2 + \omega_i^2}, \qquad p = i\omega,$$

$$k_0, k_\infty \geq 0, \qquad k_i, \omega_i > 0.$$

Let

$$W = \bigcup_{n=0}^{\infty} W_n.$$

If $Z \in W_n$, set $\nu(Z) = n$.

It is known that to every two-terminal network (a circuit with two distinguished vertices—poles) (A, φ) there corresponds uniquely (we shall denote this correspondence by Φ) a certain function from the class W; $\Phi(A, \varphi) = Z(A, \varphi)$ is its input impedance. Moreover, for any function $Z(\omega)$ from the class W_n there exists a two-terminal network (A, φ) such that $\Phi(A, \varphi) = Z(\omega)$ and such that (A, φ) has cyclomatic number equal to $n+1$, $n+2$ edges, and complexity not exceeding $2n+2$.

Lemma 1. If N is the cyclomatic number of a two-terminal network and $\tilde{n}$ is the number of its edges, then $\nu(A, \varphi) \leq N \leq \tilde{n} \leq S(A, \varphi)$. Hence

$$\nu(A, \varphi) \leq S(A, \varphi) \leq 2\nu(A, \varphi) + 2,$$

$$S(A, \varphi)/2 - 1 \leq \nu(A, \varphi) \leq S(A, \varphi). \tag{1}$$

2. The class of functions $\widetilde{W}$. Suppose that on the interval $[0, l]$ of the real axis there is given a system of differential equations for the functions $U(x)$ and $I(x)$:

$$U'(x) = -i\omega L(x) I(x), \qquad I'(x) = -i\omega C(x) U(x), \tag{2}$$

where ω is an arbitrary complex number (frequency). The functions $L(x)$ and $C(x)$ will henceforth be assumed twice continuously differentiable and

strictly positive on $[0, l]$ (such functions will be called admissible).

If the boundary condition at the end $x = l$ is given in one of the following forms: 1) $I(l) = 0$; 2) $U(l) = 0$; 3) $U(l) = Z(\omega) I(l)$, $Z(\omega) \in W$, then for system (2) the function $\widetilde{Z}(\omega) = U(0)/I(0)$ is uniquely determined. We shall also denote it by $\widetilde{W}^i(l, C(x), L(x), \omega)$, $i = 1, 2, 3$.

The class of all possible functions $\widetilde{W}^i(l, C(x), L(x), \omega)$ for system (2), with arbitrary admissible $L(x)$ and $C(x)$, arbitrary l, and boundary condition i, will be denoted by $\widetilde{W}^i$. Let $\widetilde{W}_0^i$ be the class of functions $\widetilde{Z}(\omega)$ obtained from system (2) with boundary condition i and when $L(x) = \text{const}$, $C(x) = \text{const}$. Put $\widetilde{W} = \bigcup_{i=1}^3 \widetilde{W}^i$.

As is known (^{3}), the class of functions $\widetilde{W}$ corresponds to the input impedances of nonuniform electrical lines with distributed parameters, whose current $I(x)$ and voltage $U(x)$ in the steady state are related by equations (2). In many cases, by modeling a line with distributed parameters it is sufficient to mean approximation of its input impedance by functions from the class W.

3. Complexity of modeling electrical lines with distributed parameters

Put

$$\rho^1_{a,b}(\widetilde{Z}, Z) = \max_{\omega \in [a,b]} |\widetilde{Z}(\omega) - Z(\omega)|,$$

SovietRxiv

$$\rho^2_{a,b}(\widetilde{Z}, Z) = \left(\int_b^a |\widetilde{Z}(\omega) - Z(\omega)| \, d\omega \right)^{1/2} .$$

Lemma 2. *The closure of the class W in the metric ρ^i, $i = 1, 2$, contains the class $\widetilde{W}$.*

By the complexity $S^i_{a,b}(\widetilde{Z}, \varepsilon, V)$ of ε-modeling on the interval $[a, b]$ in the metric ρ^i of a function $\widetilde{Z}(\omega) \in \widetilde{W}$ we shall mean the minimum of the complexities of all possible circuits (A, φ) with lumped parameters such that

$$\rho^i_{a,b}(\widetilde{Z}(\omega), Z(A, \varphi)) \leq \varepsilon, \qquad Z(A, \varphi) \in V \subset W. \tag{3}$$

In view of formulas (1), introduce $\nu^i_{a,b}(\widetilde{Z}, \varepsilon, V) = \min \nu(A, \varphi)$ over all circuits (A, φ) such that (3) holds. This is the quantity we shall estimate below. Note that

$$\rho^i_{a,b}(\widetilde{Z}, W_{\nu^i_{a,b}(\widetilde{Z}, \varepsilon, V)}) \leq \varepsilon,$$

but $\rho^i_{a,b}(\widetilde{Z}, W_n) > \varepsilon$ for $n < \nu^i_{a,b}(\widetilde{Z}, \varepsilon, V)$. Of interest to us will be the problem of the behavior of $\nu^i(Z, \varepsilon, W)$ for concrete functions $\widetilde{Z}(\omega)$ as $\varepsilon \to 0$.

4. Upper estimates

In the long-known and widely used method of modeling homogeneous lines (which have as input impedances functions from the class $\widetilde{W}_0$), functions $\widetilde{Z}(\omega)$ are used that are given in the form of a continued fraction of the special form:

$$Z_n = Z_{n,1} = i\omega L' + \frac{1}{i\omega C' + 1/Z_{n,2}}, \qquad Z_{n,2} = i\omega L' + \frac{1}{i\omega C' + 1/Z_{n,3}}, \ldots$$

$$\ldots \; Z_{n,n} = i\omega L' + \frac{1}{i\omega C'},$$

where $L' = \text{const}$, $C' = \text{const}$. We shall denote the class of such functions by V_0. It is not hard to verify that $Z_n \in W_n$.

Lemma 3. If $\tilde{Z} \in \widetilde{W}_0^2$, then

$$v^1_{0,\omega_0}(\tilde{Z}, \varepsilon, V_0) \sim C_1/\varepsilon^{1/2}, \quad \text{where} \quad C_1 = \omega_0 l \, [L^{3/2} C^{1/2} \operatorname{tg}(\omega_0 l (LC)^{1/2})]^{1/2}.$$

The subsequent results show that the complexity of ε-modeling in the whole class W is significantly lower.

Theorem 1. If $\tilde{Z} \in \widetilde{W}_0^1$, then for any k such that $k \geq 4\omega_0 l (LC)^{1/2}/\pi$, the following holds:

$$v^1_{0,\omega_0}(\tilde{Z},\varepsilon,W) \leq k-2+\left[\log_2\frac{1}{\varepsilon}+\log_2\left(\frac{2\omega_0 k}{\pi^2}\right)\left(\frac{L}{C}\right)^{1/2}\right]\Big/\, 2\log\left(\frac{k\pi}{2l\omega_0(LC)^{1/2}}\right).$$

Corollary 1. Under the conditions of Theorem 1,

$$v^1_{0,\omega_0}(\tilde{Z},\varepsilon,W) \leq \frac{4\omega_0 l (LC)^{1/2}}{\pi}+\frac{1}{2}\log_2\frac{1}{\varepsilon}+\frac{1}{2}\log_2\frac{l\omega_0 L}{\pi^3}.$$

Corollary 2. Under the conditions of Theorem 1,

$$v^1_{0,\omega_0}(\tilde{Z},\varepsilon,W) \leq \frac{1}{2}\log\frac{1}{\varepsilon}\Big/\log\log\frac{1}{\varepsilon}.$$

For the general case of inhomogeneous lines analogous results also hold; for example:

Theorem 2. Let $\tilde{Z}(\omega) \in \widetilde{W}^1$. In connection with equations (2), denote

$$\tau(x)=\int_0^x [L(\xi)C(\xi)]^{1/2}\,d\xi, \quad W(\tau) \leq \left[\frac{L(x)}{C(x)}\right]^{1/2},$$

$$\psi(\tau)=\left(\frac{W'}{2W}\right)^2+\left(\frac{W'}{2W}\right)', \quad b=\int_0^{\tau(l)}|\psi(t)|\,dt.$$

Let $\tilde{I}_1(\tau)$ be the solution of the equation $\tilde{I}''(\tau)=\psi(\tau)I(\tau)$ with initial conditions $\tilde{I}_1(0)=0,\ \tilde{I}_1'(0)=1$. Then for any k such that $k \geq 4\omega_0\tau(l)/\pi$, and such that for any $n > k$ the following holds:

$$1 \geq \frac{9b^2\tau^2(l)}{2n^2\pi^2}+\frac{2+3b}{n\pi}+\frac{(2b+5b^2)\tau(l)}{n\pi}+$$

$$+2\left|\int_0^{\tau(l)}\psi(t)\cos\frac{n\pi t}{\tau(l)}\cos\left[n\pi\left(1-\frac{\tau}{\tau(l)}\right)\left(1-\frac{t}{\tau(l)}\right)\right]dt\right|,$$

the following holds:

SovietRxiv

$$v^1_{0,\omega_0}(\tilde{Z},\varepsilon,W) \le k-2+\left\{\log_2\frac{1}{\varepsilon}-\log\left(1-\frac{\omega_0^2\tau^2(l)}{\pi^2(k+1/2)^2}\right)+\right.$$

$$\left.+\log\left[\frac{6\omega_0(k-1/2)\tau(l)}{\pi\tilde{I}_1(\tau(l))}\left(\frac{L(0)}{C(0)}\right)^{1/2}\right]\right\}\Big/\left\{2\log_2\frac{\pi(k+1/2)}{2\omega_0\tau(l)}\right\}.$$

5. Lower estimates. The following results show that the upper estimates of § 4 cannot be improved in order.

Theorem 3. Under the conditions of Theorem 1, the following holds:

$$v^1_{0,\omega_0}(\tilde{Z},\varepsilon,W) \ge C\log\frac{1}{\varepsilon}\Big/\log\log\frac{1}{\varepsilon}, \quad \text{where} \quad C = 1/2e^{2\pi}.$$

Theorem 4. For an arbitrary function $\tilde{Z}(\omega) \in \widetilde{W}$, the following holds:

$$v^i_{a,b}(\tilde{Z},\varepsilon,W) \sim \log\frac{1}{\varepsilon}\Big/\log\log\frac{1}{\varepsilon}, \quad i = 1,2.$$

6. Completely analogous results can be obtained for other boundary conditions, for a chain of lines with distributed parameters and included loads, for the modeling of four-terminal networks, etc.

Received
23 I 1967

REFERENCES

[1] L. D. Kudryavtsev, *UMN*, **3**, 4 (1948).
[2] V. A. Venikov, *Theory of Similarity and Modeling*, 1966.
[3] O. N. Litvinenko, V. I. Soshnikov, *Theory of Inhomogeneous Lines and Their Application in Radio Engineering*, 1964.
[4] L. I. Gutenmakher, *Electrical Models*, 1949.

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.