DISPERSION SUM RULES IN THE STATIC MODEL

PHYSICS

1967

SovietRxiv

Abstract

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UDC 530.145.1

PHYSICS

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DISPERSION SUM RULES IN THE STATIC MODEL

(Presented by Academician N. N. Bogolyubov on 26 IX 1966)

Certain assumptions concerning the high-energy behavior of the amplitude, which determine the number of subtractions in dispersion relations ([2]), have made it possible to obtain relativistic dispersion sum rules for strong interactions and meson photoproduction ([3]), subsequently generalized to the case of $SU(3)$ symmetry ([4]), and also to derive the Cabibbo–Radicati relation ([5]). In doing so, no current algebra was used.

It is of interest to find out what results are yielded by applying the method of dispersion sum rules in the static model. In particular, this is interesting from the point of view of testing the possibility of the existence of a static limit for dispersion relations.

In the present work, sum rules for the static model are obtained on the basis of assumptions about a definite high-energy behavior of the amplitude of elastic scattering of π-mesons by nucleons and of the amplitude of virtual photoproduction of π-mesons on nucleons.

§ 1. **Scattering of pions by nucleons.** As is known, in the static model it is assumed that the masses of nucleons m and isobars M are so large in comparison with the π-meson mass μ that the recoil of baryons can be neglected ([1]):

$$p_0^2 \gg \mathbf{p}^2.$$

In this case the matrix element of the meson current between two one-nucleon states has the form

$$\langle p'|j_\rho(0)|p\rangle = i\,\frac{f}{\mu}\,u^*(p')\,v(\mathbf{k}^2)\,\sigma\cdot\mathbf{k}\,\tau_\rho\,u(p), \tag{1,1}$$

where $u(p')$, $u(p)$ are nucleon spinors; f is the rationalized, renormalized, dimensionless constant; τ_ρ are isotopic Pauli matrices; $v(\mathbf{k}^2)$ is a function characterizing the cutoff in momenta and connected with the source function $\rho(x)$ by a Fourier transform:

$$v(\mathbf{k}^2) = \int e^{-i\mathbf{k}\mathbf{x}} \rho(x)\, dx. \qquad (1,2)$$

The source function is normalized so that ([1,7]): $\int \rho(x)\, dx = 1$, whence, by virtue of $(1,2)$, we obtain:

$$v(0) = 1.$$

In what follows we choose

$$v(\mathbf{k}^2) = \begin{cases} 1, & |\mathbf{k}| \leq k_{\max}, \\ 0, & |\mathbf{k}| > k_{\max}, \end{cases} \qquad (1,3)$$

i.e., the cutoff radius $R_0 \sim 1/k_{\max}$; usually $R_0 < 1/\mu,\ R_0 \sim 1/M$.

Consider a quantity of the type of an amplitude

$$T(E) = i \int e^{-iET} \Theta(t) \langle p_2 | [j_\alpha(t, 0), j_\beta(0)] | p_1 \rangle\, dt, \qquad (1,4)$$

where $j_\alpha(x)$ is the π-meson current:

$$j_\alpha(x) = i\, \frac{\delta S}{\delta \varphi_\rho(x)}\, S^+,$$

$|p_1\rangle,\ |p_2\rangle$ are one-nucleon states; α, β are isotopic states of the π-mesons.

The pion-nucleon scattering amplitude $T(E)$ may be represented in the Breit system in the form

$$T(E) = A(E) + \left\{ \frac{i\vec{\sigma} \cdot [\vec{\lambda} \times \mathbf{p}]}{p_0} - E \right\} B(E), \qquad (1,5)$$

with

$$B(E) = \delta_{\alpha\beta} B^{(odd)}(E) + \frac{1}{2} [\tau_\alpha, \tau_\beta] B^{(-)}(E), \qquad (1,6)$$

where E is the energy of the π-meson.

Let us now suppose that the amplitude $B(E)$ has such high-energy behavior that unsubtracted dispersion relations are valid for the quantities $B(E)$ and $E \cdot B(E)$, whence it follows that

$$\int_{-\infty}^{\infty} \mathrm{Im}\, B(E)\, dE = 0. \tag{1,7}$$

From the crossing invariance of the amplitude we obtain

$$B^{(odd)}(E) = -B^{(odd)*}(-E). \tag{1,8}$$

Then from (1,7) it follows that

$$\int_{0}^{\infty} \mathrm{Im}\, B^{(odd)}(E)\, dE = 0. \tag{1,9}$$

Next we approximate the amplitude by the poles of the nucleon and the isobar N^*_{33}. We use for the $N^*N\pi$-vertex the expression

$$\langle N(p)|j_\rho(0)|N^*_{pn}\rangle = \frac{f^*}{\mu}\, u^*(p')\, \tau^*_\rho\, n(p'-p)_n\, v(p'-p), \tag{1,10}$$

where Ψ_n is the wave function of the isobar; f^* is a dimensionless constant.

Then from the sum rules (1,9), with the aid of (1,1) and (1,10), we obtain

$$f^2 - f^{*2}\left[\frac{4}{9} - \frac{(M^2+m^2)(M-m)^2}{18m^2M^2}\right] = 0. \tag{1,11}$$

§ 2. **Virtual photoproduction of pions on nucleons.** As in the case of πN-scattering, we use the static model, i.e., we neglect the recoil of the nucleon. The matrix element of the electromagnetic current J between one-nucleon states has the form

$$\langle p'|\mathbf{J}(0)|p\rangle = v(\mathbf{k}^2)u^*(p')\left\{\frac{\hat{F}_e}{2m}((\mathbf{p}+\mathbf{p}') + i[\vec{\sigma}\times\mathbf{k}]) + i\hat{F}_M[\vec{\sigma}\times\mathbf{k}]\right\}u(p), \tag{2,1}$$

where

$$\mathbf{k} = \mathbf{p}' - \mathbf{p}, \qquad \hat{F}_e = \frac{1+\tau_3}{2}\,e, \qquad \hat{F}_M = \mu'^s(N) + \tau_3\mu'^v(N),$$

with $\mu'^{(s,v)}(N)$ the static anomalous isoscalar and isovector magnetic moments of the nucleon; $v(\mathbf{k}^2)$ is defined in accordance with (1,3).

Consider, as before, a quantity of the type of an amplitude:

$$F_\mu(E) = i\int e^{-iEt}\Theta(t)\langle p_2|[j_\alpha(t,0),\ J_\mu(0)]|p_1\rangle, \tag{2,2}$$

where $J_\mu(x)$ is the electromagnetic current; $|p_1\rangle, |p_2\rangle$ are one-nucleon states; $j_\alpha(x)$ is the π-meson current.

In the Breit system the amplitude $\mathbf{F}$ can be expanded in invariants ([6]):

$$\mathbf{F} = i(\vec{\sigma}\cdot\vec{\lambda})\vec{\lambda}L + i\vec{\sigma}L_1 + i(\vec{\sigma}\cdot\mathbf{p})\mathbf{p}L_2 + [\mathbf{p}\times\vec{\lambda}]L_3 + i(\vec{\sigma}\cdot\vec{\lambda})\mathbf{p}L_4 + i(\vec{\sigma}\cdot\mathbf{p})\vec{\lambda}L_5, \tag{2,3}$$

where the isotopic structure of the invariant amplitude L is as follows:

$$L = \delta_{3\alpha}L^{(v)} + \frac{1}{2}[\tau_\alpha, \tau_3]L^{(-)} + \tau_\alpha L^{(s)}, \tag{2,4}$$

where α is the isotopic state of the pion. Proceeding further analogously to the preceding case, we obtain the sum rule

$$\int_0^\infty \operatorname{Im} L^{(v)}(E)\,dE = 0. \tag{2,5}$$

For the $N^*N\pi$-vertex we use the static limit of the interaction [(3)]

$$\langle N(p')|J_\mu(0)|N^*_\nu(p)\rangle =$$

$$= i\,\frac{3\mu(N^*\to N\gamma)}{2\sqrt{2}}\,u(p')\left[\left(-\hat{k}\gamma_5 + \frac{\gamma_5(p\cdot k)}{M}\right)\delta_{\nu\mu} + \gamma_\mu k_\nu - \frac{\gamma_5 p'_\mu}{M}\right]u_\nu(p). \tag{2,6}$$

As a result, from the sum rules (2,5), using (1,1), (1,10), (2,1), (2,6), we obtain in the one-particle approximation*

$$f\cdot\mu' u(N) - \frac{f^*\mu(N^*\to N\gamma)(M+m)^2}{6\sqrt{2}M^2} = 0. \tag{2,7}$$

Let us note that the sum rules in the static model (1,11) and (2,7) are similar to the corresponding relativistic sum rules [(3)], obtained from one-dimensional dispersion relations. The difference between the static sum rules and the relativistic ones consists in the fact that in the static sum rules there are no terms depending on the meson mass and on the transferred momentum, because in the static model the latter are regarded as small in comparison with the baryon mass.

For comparison with experiment at low energies, one may calculate from the sum rule (2,7) the magnetic moment for the decay of an inclined isobar with the aid of (1,11). This gives: $\mu(N^* \to N\gamma) = \dfrac{2\sqrt{2}}{3}\, 1.29\, \mu(p)$, which agrees with the value of $\mu(N^* \to N\gamma)$ obtained from the relativistic sum rules [3], and agrees well with the experimental value

$$\mu_{\rm exp}(N^* \to N\gamma) = \frac{2\sqrt{2}}{3}(1.25 \pm 0.02)\mu(p).$$

In conclusion, one may draw the following inference: the fact that a static limit exists for the dispersion sum rules possibly indicates that a static limit exists for the dispersion relations written for πN-scattering and for virtual photoproduction of π-mesons on nucleons.

The author expresses deep gratitude to Acad. N. N. Bogoliubov for suggesting the topic and for discussions.

Joint Institute for Nuclear Research

Received
29 VIII 1966

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* We note that taking into account the interaction of π-mesons with the electromagnetic field $\mathbf{A}$

$$\mathbf{J}_M \cdot \mathbf{A} = -e(\varphi_1 \nabla \varphi_2 - \varphi_2 \nabla \varphi_1) \cdot \mathbf{A}$$

gives a contribution in the t-channel and only to the amplitudes L_2 and L_4. Analogously, taking into account the interaction

$$\mathbf{J}_I \cdot \mathbf{A} = -i\psi^* \sigma(\tau_1 \varphi_2 - \tau_2 \varphi_1)\psi \mathbf{A}$$

gives a contribution only to the invariant amplitude L_1, i.e., both of the indicated interactions do not contribute to the longitudinal amplitude L that interests us.

Note: Figure translations are in progress. See original paper for figures.

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