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NONLINEAR EFFECT OF THE FORMATION OF A SELF-SUSTAINING HIGH-TEMPERATURE ELECTRICALLY CONDUCTING GAS LAYER IN NONSTATIONARY PROCESSES OF MAGNETIC HYDRODYNAMICS

In the study of nonstationary processes of interaction of a compressible electrically conducting medium with a magnetic field, the existence was discovered of a self-sustaining, high-temperature, electrically conducting layer (a T-layer), arising as a result of the preferential release of Joule heat in a certain mass of gas. In its physical nature this phenomenon is close to the known skin effect and to phenomena of dissipative instability in magnetic hydrodynamics. However, it is not reducible to either of them and differs substantially from them. This paper considers the conditions for the occurrence and development of the phenomenon of the T-layer.

The discovered effect has a general physical character. However, we shall present some particular solutions of the equations of one-dimensional nonstationary magnetic hydrodynamics that confirm the fact of the existence of this phenomenon and illustrate its principal features.

The effect of formation of a T-layer was discovered and studied in fairly complete measure not in a physical experiment, but on a mathematical model that takes into account, in essential features, the nonlinear dependences of nonstationary processes of magnetic hydrodynamics.

1°. Self-similar problem on the expansion of an electrically conducting gas in a magnetic field. Let a non-heat-conducting gas, forming an infinitely long cylinder, be compressed in the radial direction to zero volume in such a way that a constant mass M_0 falls per unit length. The whole space is filled with a magnetic field whose lines of force are parallel to the axis of symmetry. Let, at the instant $t = 0$, an instantaneous release of energy E_0 (per unit length) take place in the gas. The motion that arises will then be self-similar with the self-similar variable $\lambda = A_0 r/t$, if the following conditions are satisfied:

a) The electrical conductivity of the gas as a function of temperature T, density ρ, and time is determined by the dependence $\sigma = \sigma_0 T^{k_0} \rho^{q_0} t^{n_*}$, where k_0 is an arbitrary number, and q_0 and n_* are related by the relation $-2q_0 + n_* + 1 = 0$.

b) A gas is considered with equations of state $P = R\rho T$, $\varepsilon = RT/(\gamma - 1)$, where P is the material pressure, ε is the internal energy, R is the gas constant, and γ is the constant ratio of specific heats.

c) The boundary conditions are prescribed on the axis of symmetry $r = 0$ in the form

$$v(0,t) = 0, \qquad \partial H/\partial r = 0 \tag{1}$$

and on the contact discontinuity $r = r_*$, bounding the gas of mass M_0,

$$P(r_*,t) = P_0/t^2, \qquad H(r_*,t) = H_0/t. \tag{2}$$

Under the indicated assumptions, the system of equations of one-dimensional nonstationary magnetohydrodynamics has the form

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial}{\partial r}\left(P + \frac{H^2}{8\pi}\right) = 0, \qquad \frac{\partial \rho}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(r\rho v),$$

$$\frac{\partial P}{\partial t} + v\frac{\partial P}{\partial r} + \frac{\gamma P}{r}\frac{\partial}{\partial r}(rv) - \frac{\gamma - 1}{4\pi\sigma}\left(\frac{\partial H}{\partial r}\right)^2 = 0, \tag{3}$$

$$\frac{\partial H}{\partial t} + v\frac{\partial H}{\partial r} + \frac{H}{r}\frac{\partial}{\partial r}(rv) - \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{\sigma}\left(\frac{\partial H}{\partial r}\right)\right) = 0$$

and is integrated in finite form. We give the resulting dependences for the functions of the magnetic-field strength, electrical conductivity, temperature, and velocity in the case $n_* = 0$ and $k_0 \neq 1/2$:

$$H = h_0(c_0 + \lambda^2)/t[c + (c_0 + \lambda^2)^2]^{1/2}, \qquad \sigma = \sigma_0\lambda^2/[c + (c_0 + \lambda^2)^2]^2 t,$$

$$T = T_0\{\lambda^2/[c + (c_0 + \lambda^2)^2]^{3/2}\}^{1/(k_0 - 0.5)}, \qquad v = \lambda. \tag{4}$$

The constants c and c_0 in formulas (4) are determined by the boundary conditions (1) and (2).

Differentiating the expressions for the function $T(\lambda)$, after a series of transformations we obtain

$$\frac{dT}{d\lambda} = \frac{3}{k_0 - 0.5} \frac{T(\lambda)}{\lambda} \left(\frac{2}{3} - \sqrt{\mathrm{Re}_m R_H} \right), \tag{5}$$

where Re_m denotes the magnetic Reynolds number ($\mathrm{Re}_m = \sigma r v$), and R_H denotes the ratio of magnetic pressure to gas pressure ($R_H = H^2/8\pi P$). In the present problem the condition $\mathrm{Re}_m R_H \leq 4$ holds.

Fixing the value of λ, and thereby determining the position of the contact discontinuity, we can, by varying the value of the parameter of magnetohydrodynamic interaction $\mathrm{Re}_m R_H$, change the position of $T_{\max}$ relative to the contact discontinuity: $T_{\max}$ will lie either at the contact discontinuity or inside the region occupied by the gas of mass M_0. Let us note one more circumstance. A comparison of the functions $T(\lambda)$ and $Q(\lambda) = ((dH/d\lambda)^2/\sigma$ shows that $T_{\max}$ lies in a region close to $Q_{\max}$, although their positions do not coincide. This is explained by the fact that the temperature depends not only on Joule dissipation, but also on how much the gas has expanded and, accordingly, cooled.

The solution obtained permits the following conclusions:

1) Expansion of a compressible electrically conducting medium in a magnetic field leads to the appearance of maxima of temperature and electrical conductivity, caused by the nonuniform distribution of Joule dissipation and by the degree of gas expansion.

2) Although relation (5) is not universal, owing to the particular form of the boundary conditions (2), it nevertheless definitely shows the decisive influence of the parameter $R_M = \sqrt{\mathrm{Re}_m R_H}$ on the character of the interaction process.

2°. Expansion of an electrically conducting gas in a magnetic field from a certain initial state. Consider a problem analogous to the preceding one, but without assuming its self-similarity. Let, at the initial moment of time $t = 0$, the electrically conducting gas form an infinitely long cylinder of radius r_0. The initial pressure in the gas, temperature, and electrical conductivity are constant, and the gas velocity is zero. All space is filled with a constant magnetic field $H = H_0$, whose lines of force are parallel to the axis of the cylinder. We shall also assume that the gas is ideal, with adiabatic exponent $\gamma = 5/3$, and that the dependence of electrical conductivity on temperature and density is determined by the formula: $\sigma = \sigma_0 T^{3/2} \rho$. At the point $r = 0$ the symmetry conditions (1) are prescribed, and at the outer boundary $r = r_0$ we have $H = H_0,\ P = 0$.

Figure 1

Figure 1: Figure 1

The solution of the above-mentioned problem was carried out numerically on a computer by the method of finite differences. System (3) was considered in mass Lagrangian variables s, where $ds = \rho r\, dr$.

Figure 1 presents the successive stages of the development of the process in time, using the functions $T(s,t)$ and $H(s,t)$ as an example. At first the outer boundary of the gas, experiencing no resistance ($\operatorname{rot} H = 0,\ P = 0$), is strongly accelerated, and the adjacent gas layers expand and cool. The arising

Fig. 1. Problem of the expansion of an electrically conducting gas in a magnetic field. Change of the temperature (**I**) and of the magnetic field H (**II**) with time. s is the mass coordinate; the time scale is $t_0 = 0.01$ sec. The numbers at the curves denote different instants of time.

motion of the gas leads to the appearance in it of induced currents of increasing intensity and to a decrease of the magnetic-field strength in the volume occupied by the gas. The maximum currents flow in the outer layers of the gas; the electrical conductivity of the outer layers of this part of the gas is lower than in the rest of the gas because of the initial cooling. This determines the most intense release of Joule heat in the outer layers of the gas, the consequence of which is a restructuring of the whole process. Beginning from some instant $t = t_0$, the decrease in the temperature of the boundary mass of gas ceases, and its growth gradually begins, while the temperature of the main mass of gas continues to decrease. Heating of the outer layer and the increase of its electrical conductivity lead to a greater effectiveness of the interaction of the gas with the magnetic field.

In this example the main feature of the process is quite evident—the self-development and self-maintenance of a high-temperature electrically conducting zone in the gas; the most essential factors in this are the dependence $\sigma = \sigma(T,\rho)$, $(\partial\sigma/\partial T > 0)$, and the compressibility of the medium. Another feature is also visible here, due to the nonlinearity of the process and distinguishing it from the ordinary linear skin effect: the heating does not begin immediately, but after the lapse of a certain critical time.

3°. Decay of a magnetohydrodynamic discontinuity in a medium with finite conductivity.

Suppose that in space filled with a gas of zero temperature and pressure, a plane piston moves with constant velocity $v = v_0$. Suppose further that the strong shock wave arising as a result of the piston motion, at the time $t = 0$, comes into contact with a uniform magnetic field $H = H_0$, whose lines of force are parallel to the plane of the shock wave. The problem that thus arises of the decay of an arbitrary discontinuity (or the problem of reflection and transmission

Fig. 2

Figure 2: Fig. 2

of a shock wave in a magnetic field) is described by the system of equations of one-dimensional nonstationary magnetohydrodynamics. The solution was found numerically by the finite-difference method. It was additionally assumed that the piston is impermeable to the magnetic field, that at the instant of "impact" of the wave on the field the distance between the shock wave and the piston was finite, and that the electrical conductivity of the gas behind the shock wave obeyed the law $\sigma = \sigma_0 T^{3/2}$.

Fig. 2. Problem of the decay of a magnetohydrodynamic discontinuity in a medium with finite conductivity. Variation of the temperature T (solid lines) and magnetic field H (dashed lines) with time. s is the mass coordinate, the time scale is $t_0 = 0.25 \cdot 10^{-4}$ sec.

Fig. 2 shows the character of the development of the process. At the point of contact of the shock wave with the magnetic field a contact discontinuity is formed; shock waves propagate to the left and to the right from it, the one to the right being magnetohydrodynamic and the one to the left gas-dynamic (as a result of weak penetration of the magnetic field). At the contact discontinuity itself there is a sharp increase in temperature due to intense Joule heat release caused by large gradients of the magnetic field.

Thus, in this process as well, a local temperature rise and the formation of a high-temperature electrically conducting layer are observed, and here the heating process begins immediately.

A local temperature rise in plasma was also observed in numerical calculations of problems of magnetohydrodynamics ([1],[2]).

However, the character of the problems considered in those works does not make it possible to identify the defining features of the T-layer as an independent physical phenomenon distinct from the skin effect.

Several examples have been considered above which illustrate the fact of the existence and the principal features of the phenomenon of the T-layer. A large number of numerical experiments, in particular taking into account nonlinear thermal conductivity and various laws $\sigma = \sigma(T, \rho)$, as well as the consideration of additional model problems, give grounds for concluding that the phenomenon of a self-sustaining high-temperature electrically conducting layer has a definite physical nature and that it can be controlled over a fairly broad range of parameters.

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Note: Figure translations are in progress. See original paper for figures.