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Abstract

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SYNTHESIS OF A MECHANICAL SYSTEM WITH A VARIATOR FOR A PRESCRIBED MOTION OF ONE OF THE LINKS

1°. In [1] it was shown that the motion of a mechanical system with a variator is described, in relative form, by an equation of the form

$$(1+z)d\tilde{\omega}/d\tau + 0.5\tilde{\omega}\, dz/d\tau = m_\partial - A\sqrt{z}, \tag{1}$$

where $z = (\xi y)^2,\ \tilde{\omega} = \omega/\omega_\text{n},\ m_\partial = M_\partial/M_\text{n},\ A = \mu/\xi$ and $\tau = t/T_\partial$.

In turn, $\xi^2 = J_2/J_1,\ y = \Omega/\omega = 1/i,\ \mu = M_c/M_\text{n}$ and $T_\partial = J_1\omega_\text{n}/M_\text{n}$, where J_1 and J_2 are the reduced moments of inertia of the driving and driven shafts; ω and Ω are the angular velocities of the driving and driven shafts; ω_n is the nominal angular velocity; M_∂ and M_c are the driving torque and the resistance torque; M_n is the nominal torque of the motor; i is the transmission ratio; t is the time of motion.

In the present work we consider problems of dynamic synthesis under conditions of maintaining a constant velocity or uniformly accelerated motion of the driving link.

2°. Let the relative angular velocity $\tilde{\omega} = \omega_1 = \text{const}$. Then equation (1) takes the form

$$dz/d\tau = 2m_\partial/\omega_1 - 2A\sqrt{z}/\omega_1. \tag{2}$$

If $\tilde{\omega} = \text{const}$, then $m_\partial = \text{const} = m_1$, since $m_\partial = f(\omega)$. The coefficient A is proportional to M_c. If the load moment depends on the angle Φ of rotation of the driven shaft, then A may be regarded as a function of τ, $\tilde{\omega}$, and z. Thus, the form of the function M_c determines the form and complexity of the solution of (2). In general, (2) has no solution. If M_c is assumed to depend linearly on time, then (2) is reduced to Abel' s equation, whose general solution, however, is also unknown.

The simplest way to solve equation (2) is to take the load as constant or, more precisely, piecewise constant, i.e. $M_c = M_1 = \text{const}$ for $0 < \tau < \tau_1$.

Fig. 1

Figure 1: Fig. 1

In this case the equation is integrated completely. Its solution has the form

$$\tau = C - \frac{\omega_1}{A^2}\left[m_1 \ln(m_1 - A\sqrt{z}) + A\sqrt{z}\right]. \tag{3}$$

Fig. 1

If the angular velocity that must be maintained constant is equal to the nominal angular velocity of the motor ω_n, then the relative angular velocity $\tilde{\omega}$ and the driving torque m_∂ are equal to unity, and the equation assumes the form

$$\tau = C - \frac{1}{A^2}\left[\ln(1 - A\sqrt{z}) + A\sqrt{z}\right]. \tag{4}$$

This law of variation of the conditional transmission ratio z is shown in Fig. 1 $(A = 0.55,\ C = 0)$.

3°. The problem of maintaining a constant angular velocity of the driven link under a varying load is solved analogously. In this case it is most convenient to ...

in this case, to use the differential equation of motion of the system written with respect to the angular velocity $\widetilde{\Omega}$ of the driven shaft ([1]). We have

$$(x + 1/A^2)d\widetilde{\Omega}/d\tau + 0.5\widetilde{\Omega}\,dx/d\tau = m_\partial\sqrt{x} - 1. \tag{5}$$

Here $x = i^2/\mu^2,\ \widetilde{\Omega} = \mu\Omega/\omega_n$.

According to the condition of the problem, the relative angular velocity of the driven shaft $\widetilde{\Omega}$ is constant and equal to $\widetilde{\Omega} = \Omega_1$. If the motor characteristic is taken to be linear and equal to $m_\partial = \gamma - (1-\gamma)\tilde{\omega}$, where γ is a constant, then under constant load equation (5) is integrable in finite form. Its solution has the form

$$\tau = C - \frac{1}{2(\gamma-1)}\ln\left|(\gamma-1)\Omega_1 x - \gamma\sqrt{x} + 1\right| - \frac{\gamma}{2(\gamma-1)\Omega_1\sqrt{\gamma^2 - 4\Omega_1(\gamma-1)}}\ln\frac{2\Omega_1(\gamma-1)\sqrt{x} - \gamma - \sqrt{\gamma^2 - 4\Omega_1(}}{2\Omega_1(\gamma-1)\sqrt{x} - \gamma + \sqrt{\gamma^2 - 4\Omega_1(}} \tag{6}$$

If the constant angular velocity of the driven link which must be kept constant is equal to the nominal one, i.e. $\widetilde{\Omega} = 1$, then (6) is somewhat simplified:

Fig. 2

Figure 2: Fig. 2

Fig. 3

Figure 3: Fig. 3

$$\tau = C = \frac{1}{2(\gamma-1)} \ln\left|(\gamma-1)x - \gamma\sqrt{x} + 1\right| - \frac{\gamma}{2(\gamma-1)(\gamma-2)} \ln \frac{\sqrt{x}-1}{\sqrt{x}-1/(\gamma-1)}. \tag{7}$$

The form of solution (7) is shown in Fig. 2 ($\gamma = 1.645,\ C = 0$).

Fig. 2

Fig. 3

4°. Let us find such a law of variation of the transmission ratio which ensures uniformly accelerated motion of the driving link. For this purpose one may use the equation

$$(J_1 + J_2 y^2) d\omega/dt + J_2 y\omega\, dy/dt = M_d - yM_c \tag{8}$$

and set in it $\omega = \varepsilon t$, where ε is the prescribed angular acceleration.

As before, the type and complexity of the solution of the differential equation for the transmission ratio y depend on the form of the functions M_d and M_c. Let us take M_c to be constant, and M_d to depend linearly on ω according to the equation $M_d = a - b\omega$. Then for y we obtain the differential equation

$$\frac{dy}{dt} = -\left[y^2 + \frac{M_c}{J_2\varepsilon} y - \frac{a - J_1\varepsilon}{J_2\varepsilon} + \frac{b}{J_2} t\right] \Big/ ty. \tag{9}$$

This is Abel' s equation, for which the general solution is unknown. The qualitative investigation carried out for equation (9) showed that, in the range of variation of the variables under consideration ($t > 0, y > 0$), the equation has two singular points A and B (Fig. 3). The singular point A is of saddle type, and the singular point B is of focus type. Their coordinates are

$$(A).\ t = 0, \qquad y_0 = \left[-M_c + \sqrt{M_c^2 + 4(a - J_1\varepsilon)J_2\varepsilon}\right] / 2J_2\varepsilon$$

$$(B).\ t = (a - J_1\varepsilon)/b\varepsilon, \qquad y = 0. \tag{10}$$

From consideration of the neighborhood of the singular point A, one may conclude that only two curves—the separatrices—pass through the point A, one of which is the ordinate axis. Since near the point A there are no other singular points, and integral curves can intersect only at singular points, this means that the ordinate axis is intersected by only one curve—the other separatrix—and no other integral curve can intersect the vertical ordinate axis.

From all this one may conclude that the conditions of the posed problem can be satisfied by only one law of variation of the transmission ratio—with initial point A, having coordinates $(0; y_0)$, and being the separatrix AB. Moreover, if at the initial instant the variator has a transmission ratio different from y_0, then, in order that the driving shaft move with the prescribed constant acceleration ε, it is necessary instantaneously to change the transmission ratio of the variator to the value y_0 and then to vary it along the separatrix AB.

Physically this means that there exists only one initial transmission ratio y_0 for which the driving moment M_d and the resistance moment M_c, reduced to the driving shaft, acting on the mass of the driving link plus the mass of the driven link reduced to the driver, force the driving shaft to move with the prescribed acceleration ε.

It should be noted that uniformly accelerated motion is the fastest motion when the maximum value of the acceleration is limited. The system under consideration, therefore, moving uniformly accelerated, will accelerate from rest to a prescribed angular velocity in the minimum time. Thus it may be said that in this work a law of variation of the transmission ratio has been found which synthesizes a system optimal with respect to the speed of response of the driving link.

As the prescribed acceleration of the system ε is increased, the initial value of the transmission ratio y_0, as follows from (10), decreases.

For $\varepsilon = a/J_1$, solution (14) degenerates into $y \equiv 0$, i.e., the driven part is in fact disconnected, and the motor accelerates itself. The driven shaft then remains at rest.

As was noted earlier, the exact solution of (9) is unknown. However, an approximate analytical solution can be found, for example by the method of successive approximations. Thus, if as the initial approximation one chooses a quadratic parabola having contact of the 2nd order with the initial point, of the form

$$y = kt^2 + lt + m,$$

where

$$m = \left[\sqrt{M_c^2 + 4(a - J_1\varepsilon)J_2\varepsilon} - M_c\right] / 2J_2\varepsilon,$$

$$l = -b\varepsilon/(3mJ_2\varepsilon + M_c), \qquad k = 2l^2 J_2\varepsilon/(4mJ_2\varepsilon + M_c),$$

then the desired approximate law of variation of the transmission ratio has the form

$$y = m - lt - \frac{k}{2}t^2 - \frac{mJ_2\varepsilon + M_c}{2J_2\varepsilon} \ln\left|\frac{kt^2 + lt + m}{m}\right| - \frac{l(mJ_2\varepsilon + M_c) + 2b\varepsilon}{2J_2\varepsilon\sqrt{l^2 - 4km}} \ln\left|\frac{2m + (l + \sqrt{l^2 - 4km})t}{2m + (l - \sqrt{l^2 - 4km})t}\right|.$$

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1. I. I. Artobolevskii, V. A. Zinov' ev, N. V. Umnov, DAN, **173**, No. 5 (1967).

Note: Figure translations are in progress. See original paper for figures.