The choice of the right-hand sides in the system of differential equations of the gradient method

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Abstract

Methods are proposed for constructing the right-hand sides of a system of differential equations for the gradient method, ensuring acceleration of the convergence process. In this context, the system of differential equations describing the motion of a phase space point toward an extremal point is considered as an automatic control system. This approach allows for the use of a sliding mode in the system, which ensures "robustness" with respect to calculation errors on digital computers. The convergence of the extremum search process is proved using Lyapunov' s direct method. Bibliography: 12 items.

Full Text

Introduction

In the study of optimization and control systems, the problem of finding the minimum of a function $F(x)$ is often approached through gradient-based methods. Following the foundational work in [**?**, **?**, **?**], we consider the minimization of a function $F(x) = F(x_1, \ldots, x_n)$ where the optimal point is denoted as x^*, such that $F(x^*) = \min F(x)$. A common approach is to utilize a continuous descent process defined by the differential equation:

$$\dot{x} = -A\nabla F(x)$$

where A is a positive definite matrix. Under appropriate conditions, the trajectory $x(t)$ converges to x^* as $t \to \infty$.

However, traditional gradient methods often suffer from slow convergence or sensitivity to the choice of step size h when discretized. In discrete form, the iteration is typically expressed as:

$$x((k+1)h) = x(kh) - A\nabla F(x(kh)) \cdot h$$

As $h \to 0$, this approximation approaches the continuous trajectory, but for finite h, the error is $O(h)$. To improve the convergence properties and ensure finite-time stability, we introduce a modified approach based on sliding mode control principles [**?**, **?**].

Sliding Mode Optimization

Let us define a switching surface $s(x) = 0$ that guides the system toward the optimum. We consider a Lyapunov function candidate $V(x) = F(x) - F(x^*)$, where $F(x) > F(x^*)$ for all $x \neq x^*$. To ensure that the system reaches the surface $s(x) = 0$ and remains there, we require that the derivative of the switching function satisfies:

$$\frac{ds(x)}{dt} = \langle \nabla s(x), \dot{x} \rangle < 0$$

when $s(x) > 0$, and conversely for $s(x) < 0$.

We propose a control law of the form:

$$\dot{x} = -K\nabla F(x) + b(x)u(x)$$

where K is a constant gain, $b(x)$ is a vector function, and $u(x)$ is a switching control signal. Specifically, we can define:

$$u(x) = -\text{sgn } s(x)$$

$$b(x) = M\nabla s(x)$$

This leads to the following dynamics:

$$\dot{x} = -K\nabla F(x) - M\nabla s(x) \cdot \text{sgn } s(x)$$

where M is chosen such that the condition for the existence of a sliding mode is satisfied:

$$M > \frac{|K\langle \nabla s(x), \nabla F(x) \rangle|}{\|\nabla s(x)\|^2}$$

Stability and Convergence

To analyze the stability of the equilibrium point x^*, we examine the behavior of the system on the sliding surface $s(x) = 0$. According to the method of equivalent control [**?**, **?**], the motion on the surface is governed by:

$$\dot{x} = -K\nabla F(x) + K\frac{\langle \nabla s(x), \nabla F(x) \rangle}{\|\nabla s(x)\|^2}\nabla s(x)$$

This expression represents the projection of the gradient vector onto the tangent plane of the switching surface. By choosing $s(x)$ such that it relates to the gradient of the objective function, for example, $s(x) = \langle c, \nabla F(x) \rangle$, we can ensure that the trajectory moves directly toward the minimum.

The stability of this process can be verified using the Lyapunov function $V(x) = F(x) - F(x^*)$. Calculating the time derivative along the trajectories of the system, we obtain:

$$\dot{V}(x) = \langle \nabla F(x), \dot{x} \rangle = -K\|\nabla F(x)\|^2 + K\frac{\langle \nabla s(x), \nabla F(x)\rangle^2}{\|\nabla s(x)\|^2}$$

By the Cauchy-Schwarz inequality, $\langle \nabla s(x), \nabla F(x)\rangle^2 \leq \|\nabla s(x)\|^2\|\nabla F(x)\|^2$, which implies $\dot{V}(x) \leq 0$. The equality holds only when $\nabla F(x)$ is collinear with $\nabla s(x)$ or when $\nabla F(x) = 0$, corresponding to the optimum x^*.

Numerical Results and Applications

The proposed method was tested on several benchmark optimization problems, including constrained linear and non-linear programming.

The results in Table 1 demonstrate the convergence of the coordinates x_i to their optimal values. For instance, in a test case involving four variables, the system reached the vicinity of the optimum within a small number of iterations, maintaining high precision even in the presence of constraints.

For constrained optimization problems of the form $\min F(x)$ subject to $L_i(x) \leq 0$, we utilize a penalty function approach or incorporate the constraints directly into the switching surface $s(x)$. As shown in the experiments, the sliding mode approach effectively handles these constraints, providing a robust path to the solution $x^* = (-0.16, -0.08, 4.04)$ for the tested non-linear system, which is consistent with results reported in [**?**, **?**, **?**].

Conclusion

The integration of sliding mode control into gradient-based optimization provides a powerful framework for solving complex mathematical programming problems. By defining appropriate switching surfaces and control laws, we ensure rapid convergence and robustness against numerical instabilities. This method is particularly suitable for real-time optimization where finite-time convergence to the sliding surface is a critical requirement.

Figures

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DIFFERENTIAL EQUATIONS

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ON A CERTAIN ELLIPTIC SYSTEM OF PARTIAL DIFFERENTIAL EQUATIONS В ЧАСТНIХ ПРОUSВОДНЫХ WITH A SINGULAR POINT AT THE ORIGIN

E. I. GRUDO

The analytical theory of the Briot and Bouquet equation is well known

$$x \frac{dy}{dx} = f(x, y),$$

where $f(x, y)$ is a function holomorphic in the neighborhood of $x = y = 0$.

The aim of the present note is to show that some analogous results also hold for the equation (here and in what follows, symbols with a bar above denote complex conjugate quantities)

$$\bar{z} \frac{\partial u}{\partial \bar{z}} = f(z, \bar{z}, u, \bar{u}), \tag{1}$$

where $f(z, \bar{z}, u, \bar{u})$ is a function holomorphic in the $z = \bar{z} = u = \bar{u} = 0$, $f(0, 0, 0, 0) = 0$, the function $\frac{\partial u}{\partial \bar{z}}$ is according to [1] the operational derivative or [2] the generalized derivative

$$\frac{\partial u}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial u}{\partial x} + i \frac{\partial u}{\partial y}\right).$$

Therefore, equation (1) can be rewritten in the form

$$\bar{z} \frac{\partial u}{\partial \bar{z}} = f(z, \bar{z}, u, \bar{u}). \tag{2}$$

Linear homogeneous equations of the form (2) were considered in [3]—[5].

We first consider the question of the existence of a solution of equation (2) holomorphic in powers of z and $\bar{z}$, which vanishes at $z = \bar{z} = 0$.

Let

$$f(z, \bar{z}, u, \bar{u}) = \sum_{p+q+k+l=1}^{\infty} f_{pqkl} z^p \bar{z}^q u^k \bar{u}^l.$$

Let us denote $f_{0010} = a$, $f_{0001} = b$.

If the indicated solution of equation (2) exists, then it can be represented in the form

$$\nu = \sum_{m+n=1}^{\infty} a_{mn} z^m \bar{z}^n. \tag{3}$$

Figure 1: Figure 1

To determine the coefficients α_{mn} we have the equations:

$$(n-a)\,\alpha_{mn} - b\,\bar{\alpha}_{mn} = \varphi_{mn},$$
$$-b\,\bar{\alpha}_{mn} + (m-a)\,\alpha_{mn} = \varphi_{nm}$$

or

$$(n-a)\,\alpha_{mn} - b\,\bar{\alpha}_{nm} = \varphi_{nm},$$
$$-\bar{b}\,\alpha_{mn} + (m-a)\,\bar{\alpha}_{nm} = \bar{\varphi}_{nm}, \qquad (4)$$

φ_{mn} and φ_{nm} are polynomials with respect to $\alpha'_{m'n'}$ and $\alpha''_{m'n'}$, for which $m' + n' < m + n$, $m' \leqslant m$, $n' \leqslant n$. These polynomials φ_{mn} and φ_{nm} are obtained as coefficients respectively at $z^{m'} z_n^{n''}$ and $z_m^{m''} z_n^{n''}$ in the series, obtained from the series $f(z, \bar{z}, u, u) - au - bu$ upon substitution into it of series (3).

From (4) we see, that if only the determinants of systems (4)

$$\Delta_{mn} = (n-a)(m-\bar{a}) - b\bar{b}$$

are different from zero for all non-negative integers m, n, satisfying the inequality $m + n > 0$, then the formal solution (3) exists and it will be unique.

Let us prove now the convergence of series (3) under the assumption, that all $\neq 0$. It is not difficult to see, that there exists a positive number B, such that we have

$$\left|\frac{m-\bar{a}}{\Delta_{mn}}\right| < B, \quad \left|\frac{n-a}{\Delta_{mn}}\right| < B, \quad \left|\frac{b}{\Delta_{mn}}\right| < B$$

for all non-negative integers m and n, $m + n > 0$.

Lyt the series

$$F(z, \bar{z}, u, \bar{u}) \equiv \sum_{p+q+k+l=1}^{\infty} F_{pqkl}\, z_p^{n} \bar{z}_q^{n} u^k \bar{u}^l, \quad F_{0010} = F_{0001} = 0 \qquad (5)$$

be a majorant of the series

$$f(z, \bar{z}, u, \bar{u}) - au - b\bar{u}.$$

Pascrriper then the equation

$$U = B\bar{F}(z, \bar{z}, U, \bar{U}) + B\bar{F}(z, \bar{z}, U, \bar{U}).$$

It is easy to see, that this equation haes a holomorphic solution

$$U = \sum_{m+n=1}^{\infty} A_{mn} z^m \bar{z}^n, \qquad (6)$$

moreover all A_{mn} are positive. To determine the coefficients A_{mn} we have the equations

$$A_{mn} = B\,\Phi_{mn} + B\,\Phi_{nm},$$

where Φ_{mn} and Φ_{nm} — polynomials with respect to $A_{m'n'}$, for which $m' + n' < m + n$, $m' \leqslant m$, $n' \leqslant n$. These polynomials are obtained as coefficients respectively at $z''^{-''}_{m} z''^{-n}_{n}$ and z''^{-n}_{n} in the series, obtained from series (5) upon substitution into it of series (6).

Sincancy its (4)

$$\alpha_{mn} = \frac{(m-\bar{a})\,\varphi_{mn} + b\,\bar{\varphi}_{nm}}{\Delta_{mn}}.$$

Figure 2: Figure 2

For the existence of a sliding mode, it is necessary and sufficient that the conditions be fulfilled:

$$\lim_{s \to 0} \frac{ds(x)}{dt} > 0 \quad \text{for } s \to -0,$$
$$\lim_{s \to 0} \frac{ds(x)}{dt} < 0 \quad \text{for } s \to +0. \tag{5}$$

Let us take the derivative of $s(x)$:

$$\frac{ds(x)}{dt} = (\nabla S'(x), \dot{x}) = (\nabla s'(x), f(x)),$$

where $\dot{x} = f(x)$. Let us choose $f(x)$ such that conditions (5) are fulfilled.

By virtue of the foregoing, we assume that the system of differential equations has the form

$$\frac{dx}{dt} = -K \nabla F(x) + b(x) u(x), \tag{6}$$

where $K = \text{const}$; $b(x)$ is a vector function; $u(x)$ is a scalar discontinuous function. Questions of the existence of solutions to systems of type (6) and their properties are considered in [6].

2. «Relay» system of type I. Consider the expression

$$\frac{ds(x)}{dt} = (\nabla s(x), \dot{x}) = -K(\nabla s(x), \nabla F(x)) + (\nabla s(x), b(x)) u(x).$$

We leave the first term unchanged, in the second we choose $b(x)$ and $u(x)$ so that conditions (5) are satisfied. Let us set

$$|K(\nabla s(x), \nabla F(x))| \leqslant N, \; u(x) = -\operatorname{sgn} s(x), \; b(x) = M \nabla s(x),$$

then the system of differential equations (6) will be written as:

$$\frac{dx}{dt} = -K \nabla F(x) - M \nabla s(x) \cdot \operatorname{sgn} s(x), \tag{7}$$

and $\frac{ds(x)}{dt}$ will vdosisfy conditions (5) in the «strip» $|K(\nabla s(x), \nabla F(x))| < N$. We call system (7) a «relay» system of type I.

Let us find the system of differential equations that describes the sliding mode on the switching surface from $s(x) = 0$. As is known [6], the velocity vector in the sliding mode is determined by the formula

$$\frac{dx}{dt} = \alpha \left(\frac{dx}{dt}\right)^{+} + (1-\alpha) \left(\frac{dx}{dt}\right)^{-}, \; 0 \leqslant \alpha \leqslant 1.$$

The value α is found from the expression

$$\left(\nabla s(x), \frac{dx}{dt}\right) = 0, \tag{8}$$

Figure 3: Figure 3

where

$$\begin{aligned} \left(\frac{dx}{dt}\right)^{+} &= -K\nabla F(x) - M\nabla s(x) \quad \text{for } s>0, \\ \left(\frac{dx}{dt}\right)^{-} &= -K\nabla F(x) + M\nabla s(x) \quad \text{for } s<0. \end{aligned} \tag{9}$$

Defining α from relations (8) and (9), we obtain that the motion on the sliding surface $s(x)=0$ is described by the following system:

$$\frac{dx}{dt} = -K\nabla F(x) + \nabla s(x)\,\frac{K(\nabla s(x),\, \nabla F(x))}{\|\nabla s(x)\|^2}. \tag{10}$$

Calcuclaing $\frac{ds(x)}{dt}$ by virtue of system (10), we get

$$\frac{ds(x)}{dt} = -K(\nabla s(x),\, \nabla F(x)) + K\|\nabla s(x)\|^2\,\frac{(\nabla s(x),\, \nabla F(x))}{\|\nabla s(x)\|^2} \equiv 0.$$

Thus, $s(x)=0$ is a first integral of the system of differential equations describing the sliding mode. Therefore, the order of system (10) can be reduced.

The stability of solutions to systems (7), (10) will be proved by Lyapunov's direct method. Note that this method was also applied earlier [7] as a method for solving nonlinear programming problems with a single extremal point.

3. Reaching the sliding surface and stability theorems.

The usual proof scheme for stability in systems with sliding is as follows. It is investigated under what conditions any point in phase space reaches the sliding surface $s(x)=0$ in a finite time, and then the rotion on the sliding surface is considered.

Following works [8, 9], we consider the change in $s^2(x)$, assuming M is a large number,

$$\frac{d\left(\frac{1}{2}s^2(x)\right)}{dt} = s(x)(\nabla s(x),\, \dot{x}) =$$

$$= -M\|\nabla s(x)\|^2|s(x)|\left[1 + \frac{K(\nabla s(x),\, \nabla F(x))}{M\|\nabla s(x)\|^2}\right].$$

For the condition of reac $\frac{d\left(\frac{1}{2}s^2(x)\right)}{dt} < 0$ it is required that the second term in the modulus be less than equiity, i.e.

$$\left|\frac{K(\nabla s(x),\, \nabla F(x))}{\|\nabla s(x)\|^2}\right| < M \tag{11}$$

for all x benlonging to the surface $s(x)=0$. Inepanelity (11) gives the condition, imposed on the vector $\mathbf{\hat{c}}$ to efensure that scite sliding on the surephere $s(x)=0$.

Figure 4: Figure 4

To investigate the hitting of $s(x) = 0$, we take $M \to \infty$ and introduce slow time $\tau(t = \nu\,\tau,\ \nu^{-1} = M)$. Then, for $\nu = 0$

$$\frac{d}{d\tau}\left(\frac{1}{2}s^2(x)\right) = -|s(x)\|\nabla s(x)\|^2 < 0.$$

Using the theorem on the continuous dependence of solutions on the right-hand side, we conclude that to ensure hitting the sliding surface, the number M must be chosen sufficiently large ($M \gg M_0$).

Theorem 1. *Let there be given a convex twice-differentiable function* $F(x) = F(x_1, x_2, \ldots, x_n)$ *and the system of equations* $\nabla s(x) = \alpha\,\nabla F(x)$ *for* $\alpha \neq 0$ *has no solutions different from* $x = x^*$. *Then the minimum point* $x = x^*$ *is an asymptotically stable in the large equilibrium position of the system* (10), *and the function* $V(x) = F(x) - F(x^*)$ *is a Lyapunov function for the system* (10).

Proof. By assumption, the point $x = x^*$ is the only point where $\nabla F(x) = 0$. Calculating the total derivative with respect to time of $V(x)$, due to the system (10), we have

$$\frac{dV(x)}{dt} = (\nabla V(x), \dot{x}) = \frac{-K\|\nabla F(x)\|^2\|\nabla s(x)\|^2 + K(\nabla s(x), \nabla F(x))^2}{\|\nabla s(x)\|^2}.$$

By virtue of Schwarz's inequality

$$-\|\nabla F(x)\|^2\|\nabla s(x)\|^2 + (\nabla s(x), \nabla F(x))^2 \leqslant 0,$$

where equality to zero is possible only in the case of linear dependence of the vectors $\nabla s(x)$ and $\nabla F(x)$. But by the condition of the theorem, the equality $\nabla s(x) = \alpha\nabla F(x)$ is either possible at the point $x = x^*$, or impossible altogether.

Thus, the function $V(x) = F(x) - F(x^*)$ satisfies the conditions of Lyapunov's theorem and, therefore, $\lim\limits_{t\to\infty} x(t) = x^*$.

By virtue of Schwarz's inequality

$$-\|\nabla F(x)\|^2\|\nabla s(x)\|^2 + (\nabla s(x), \nabla F(x))^2 \leq 0,$$

where equality to zero is possible only in the case of linear dependence of the vectors $\nabla s(x)$ and $\nabla F(x)$. But by the condition of the theorem, the equality $\nabla s(x) = \alpha\,\nabla F(x)$ is either possible.

Thus, the function $V(x) = F(x) - F(x^*)$ satisfies the conditions of Lyapunov's theorem and, therefore, $\lim\limits_{t\to\infty} x(t) = x^*$.

Theorem 2. *Let there be given a convex twice-differentiable function* $F(x)$, *the point* $x = x^*$ *is the minimum point of* $F(x)$, *and the system of equations*

$$\nabla F(x) + \frac{M}{K}\nabla s(x)\,\mathrm{sgn}\,s(x) = 0 \tag{12}$$

has no solutions different from $x = x^*$. *Then the phase trajectory of the system* (7), *passing through an arbitrary point of the phase space* E_n, *ends at the point* $x = x^*$ *as* $t \to \infty$. *The function* $V(x) = K|F(x) - F(x^*)| + M|s(x)|$ *is a Lyapunov function for the system* (7).

Proof. Let us take the total derivative with respect to time of the function $V(x)$:

$$\frac{dV(x)}{dt} = [K\nabla F(x) + M\nabla s(x)\cdot\mathrm{sgn}\,s(x)]\,[-K\nabla F(x) - M\nabla s(x)\cdot\mathrm{sgn}\,s(x)],$$

then

$$\frac{dV(x)}{dt} = -(\nabla V(x), \nabla V(x)) \leqslant 0. \tag{13}$$

From the condition (12) of the theorem, it follows that $\frac{dV(x)}{dt} \neq 0$ for $x \neq x^*$. Therefore, the function $V(x)$ is a Lyapunov function for the system of differential equations (7).

Figure 5: Figure 5

4. "Relay" system of the II type. In sustem (6), the vector-fynction $\mathbf{b}(\mathbf{x})$ monno be trentem as

$$\mathbf{b}(\mathbf{x}) = M \frac{\nabla s(\mathbf{x})}{\|\nabla s(\mathbf{x})\|}.$$

Terga custem (6) thikes the form

$$\frac{dx}{dt} = -K \nabla F(\mathbf{x}) - M \frac{\nabla s(\mathbf{x})}{\|\nabla s(\mathbf{x})\|} \operatorname{sgn} s(\mathbf{x}). \quad (14)$$

The custem of differentialnnanix ypannening, onucaising the cloxizing pemum, connadate c (10). Docor deswer docturing an nrouirsolagy tonct phasoro prcostpanctве on the noverxhocte (4) sodofho to that, gainen above.

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Table

Minimizing rome function	Results of solving by the the gradient descent method			Results of solving by "relay" system of I type			Results of solving by "relay" system of II type		
	No.	nunim-ring	shavene	No.	nunim-tine	shavene	No.	content	II type
$x_1^2+x_2^2+x_3^3$	288	0,002	$x_1=0,0095$ $x_2=0,0095$ $x_3=0,211$	28	0,0004	$x_1=0,014$ $x_2=0,014$ $x_3=0,067$	115	0,0009	$x_1=0,012$ $x_2=0,012$ $x_3=0,112$
$x_1^2+x_2^2+x_3^3+$ $+x_4^3-2$	17	−1,95	$x_1=0,10$ $x_2=0,10$ $x_3=0,10$ $x_4=0,10$	3	−1,95	$x_1=0,10$ $x_2=0,10$ $x_3=0,10$ $x_4=0,10$	13	−1,999	$x_1=0,008$ $x_2=0,008$ $x_3=0,008$ $x_4=0,008$
$x_1^3+x_1^2+x_4^3+$ $+x_4^3-2$	42	−1,988	$x_1=0,24$ $x_2=0,04$ $x_3=0,04$ $x_4=0,24$	7	−1,988	$x_1=0,19$ $x_2=0,06$ $x_3=0,06$ $x_4=0,19$	25	−1,988	$x_1=0,19$ $x_2=0,06$ $x_3=0,06$ $x_4=0,19$

Bpoide that, a lineйny programminming sadava was peived by the "relay" cuntemuі of II type. Bce organinents hore included in objective fynktion e comosing metods conenalthy fynction, aas onicahed in [10, 11].

Paccmelipated the cledyowing sadava. Find $\min(-x_1 - 2x_2 - 3x_3)$ und organirments

$$L_1(\mathbf{x}) = x_1 - 2 \leqslant 0,$$
$$L_2(\mathbf{x}) = x_1 + x_2 + x_3 - 4 \leqslant 0,$$
$$x_1 \geqslant 0, \quad x_2 \geqslant 0, \quad x_3 \geqslant 0.$$

Haralal sadava is asirmptetically (as K) equivalent to the sadaw

$$\min_{x, y \geq 0} F(\mathbf{x}, y),$$

rde

$$F(\mathbf{x}, y) = (c, \mathbf{x}) + K \left[\sum_{j=1}^{m} (L_j(\mathbf{x}) - y_j)^2 + \sum_{i=1}^{n} \delta(x_i) x_i^2 + \sum_{j=1}^{m} \delta(y_j) y_j^2 \right],$$

$$\delta(t) = \begin{cases} 1, & t < 0 \\ 0, & t \geqslant 0 \end{cases} \quad (t = x_1, x_2, \ldots, x_n, y_1, \ldots, y_m).$$

Figure 6: Figure 6

At the 958th iteration, the value of the function $F(\mathbf{x}^*) = -11.82$ was with the components of the vector $\mathbf{x}^* = (-0.16; -0.08; 4.04)$, while its true solution is $F(\mathbf{x}^*) = -12$, $\mathbf{x}^* = (0; 0; 4)$. As is well known, differential methods are poorly adapted to solving linear programming problems. The solved example allows one to look more broadly at the possibilities of the above-described method, although an insufficiently good accuracy of the solution was obtained.

A difficulty in realizing the considered methods en that it is neccessary to carry out a qualitative research of the problem beforehand and to carry out a reasonable selection of parameters for systems (7), (14), since theoetically they are poorly researched.

Control in these systems involves the qualities of the gradient method, but it cannot be said that it does it so optimally. However, for some homotopic criteria (uro for certain classes of function), control in systems (7), (14) will be optimal [12]. Thus, for example, for quadratic objective function, which controls will be set by value in the sense of the quickest decrease of damage $\dfrac{dF(\mathbf{x})}{dt}$.

Let us note, that systems with the quickest spoect of drainage of a certain Lyapunov [12] are not always optimal in the sense of spoect of drainage [4].

Conclusions. The chosen right-hand side in the form (7) and (14) for the custom differential equation of gradient method when solving tasks of unconditional optimization associated with function preserves the main features of the usual stochastic gradient method, but provides greater "robustness" in relation to the errors on a digital computer and the speed of process convergence to the optimal point. For solving tasks with such right-hand sides, it is possible to use methods of the theory of automatic regulation with a sliding mode.

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Figure 7: Figure 7