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Abstract

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SOME PARTICULAR SOLUTIONS OF THE EQUATIONS DESCRIBING AXISYMMETRIC MOTIONS OF A GRAVITATING RAREFIED PLASMA

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Consider axisymmetric nonstationary motions of a rarefied plasma in the presence of a magnetic field and self-gravitation. We shall assume here that the magnetic-field intensity vector **H** is perpendicular to the trajectories of the particles. Then the magnetic lines of force may be: 1) either straight lines parallel to the axis of symmetry; 2) or concentric circles with centers on the axis of symmetry; 3) or helical lines.

If dissipative processes are absent, then the equations describing such motions (with allowance for pressure anisotropy) may be written in the form

$$\rho\frac{dv_r}{dt}+\frac{\partial}{\partial r}(p_\perp+h)+\frac{1}{r}\left[(p_\perp-p_\parallel)\frac{h_\varphi}{h}+2(h_\varphi+Gm\rho)\right]=0,$$

$$\frac{d\rho}{dt}+\rho\left(\frac{\partial v_r}{\partial r}+\frac{v_r}{r}\right)=0,\qquad \frac{\partial m}{\partial r}-2\pi\rho r=0,\tag{1}$$

$$\frac{d}{dt}\left(\frac{h_z}{\rho^2}\right)=0,\qquad \frac{d}{dt}\left(\frac{h_\varphi}{\rho^2 r}\right)=0,\qquad \frac{d}{dt}\left(\frac{p_\perp}{\rho h^{1/2}}\right)=0,\qquad \frac{d}{dt}\left(\frac{p_\parallel h}{\rho^3}\right)=0.$$

Here r is the distance from the particle to the axis of symmetry; t is time; v_r is the radial velocity; ρ is the density; m is the mass; $h=h_z+h_\varphi$; $h_z=H_z^2/8\pi$; $h_\varphi=H_\varphi^2/8\pi$; H_z and H_φ are the axial and transverse components of the magnetic-field intensity vector; G is the gravitational constant; $p_\perp$ and $p_\parallel$ are the pressures, respectively perpendicular and parallel to the magnetic field (transverse and longitudinal pressures).

In case 1) $h_z\neq 0,\ h_\varphi=0$; in case 2) $h_z=0,\ h_\varphi\neq 0$; in case 3) $h_z\neq 0,\ h_\varphi\neq 0$.

In the absence of gravitational forces, equations analogous to equations (1) were obtained earlier in works (^{1,2}). In work (^{2}) some exact solutions of system (1) for $G=0$ were found.

If we assume that the dependence of the radial velocity of a particle on r and t has the form

$$v_r = \frac{r}{\mu(t)} \frac{d\mu}{dt}, \tag{2}$$

where $\mu(t)$ is a function of time (to be determined), then by direct verification it is easy to confirm the existence of the following particular exact solutions of system (1).

I.

$$\begin{gathered} \rho = \rho_0 \mu^{-2}, \qquad h_z = \left[{}^1/_2 b_3 \rho_0 \xi^2 + b_2 - P(\xi)\right] \mu^{-4}, \\ h_\varphi = 0, \qquad p_\perp = P(\xi)\mu^{-4}, \qquad p_\parallel = Q(\xi)\mu^{-2}. \end{gathered} \tag{3}$$

Here $P(\xi)$ and $Q(\xi)$ are arbitrary functions of the Lagrangian coordinate $\xi = r/\mu$. The dependence $\mu(t)$ is found from the differential equation:

$$\left(\frac{d\mu}{dt}\right)^2 = b_3 \mu^{-2} - 2b_1 \ln\mu + b_4 = f_1(\mu); \tag{3'}$$

$b_1, \ldots, b_4$ are arbitrary constants, $\rho_0 = b_1/2\pi G$.

II.

$$\rho = \frac{R'}{r\mu} = \frac{T'}{r^2}, \quad h_z = 0, \quad h_\varphi = \frac{1}{r^2}\left[a_4 + a_5(\xi T - \Pi_1) - 2\pi G R^2\right],$$

$$p_\perp = \frac{1}{r\mu^2}(a_2 + a_3 T), \quad p_\parallel = \frac{a_1}{\mu^4} T',$$

$$\left(\frac{d\mu}{dt}\right)^2 = 2a_3\mu^{-1} - a_1\mu^{-2} - 2a_5 \ln\mu + a_6 = f_2(\mu). \tag{4}$$

Here $R(\xi)$ is an arbitrary function; $a_1, \ldots, a_6$ are arbitrary constants; the functions $\Pi_1(\xi)$ and $T(\xi)$ are related to the function $R(\xi)$ by the relation

$$T' = \xi R' = \Pi_1''$$

(the prime denotes differentiation with respect to ξ).

III.

$$\rho = \frac{R'}{r\mu}, \quad h_z = \frac{k^2}{r^2\mu^2}\Phi(\xi), \quad h_\varphi = \frac{1}{r^2}\Phi(\xi),$$

$$p_\perp = \frac{c_9}{\alpha}\frac{(\mu^2 + k^2)^{1/2}}{\mu^4}\,\xi R', \quad p_\parallel = \frac{c_3}{\mu^2(\mu^2 + k^2)}\,\xi R',$$

$$\left(\frac{d\mu}{dt}\right)^2 = c_1\mu^{-2} - c_3(\mu^2+k^2)^{-1} - 2c_5 \ln\mu + c_9\mu^{-2}(\mu^2+k^2)^{1/2} + \tag{5}$$

$$+\frac{c_9}{k}\frac{2+\alpha}{\alpha}\ln\frac{k+(\mu^2+k^2)^{1/2}}{\mu} + c_{11},$$

$$R(\xi) = b\xi^\alpha - \beta, \quad \Phi(\xi) = A - 2\pi G R^2 + c_5\int R'\xi^2\,d\xi.$$

Here $A = 2\pi G\beta^2$; α can take only two values ($\alpha = 1, \alpha = 2$). In this case $\beta = 0$, $3c_1 = k^2 c_5$, if $\alpha = 1$, and $2c_1 = k^2(c_5 - 4\pi G b)$, if $\alpha = 2$; $c_1, c_3, c_5, c_9, c_{11}, \beta, b, k^2$ are constants.

Let us note that, for the magnetohydrodynamic equations, solutions analogous to solutions I—III were found by A. G. Kulikovskii (^{3}), I. M. Yavorskaya (^{4}), McVittie (^{5}), and the author (^{6}).

Solutions I—III can be generalized to the case when the plasma particles have, in addition to the radial velocity v_r, also a rotational velocity v_φ, determined from the formula

$$v_\varphi^2 = \xi\chi'(\xi)\mu^{-2}. \tag{6}$$

A similar generalization was made earlier by Yu. P. Ladikov (^{7}) for magnetic hydrodynamics with isotropic pressure and by V. P. Korobeinikov (^{2}) for the hydrodynamics of a rarefied plasma without taking gravitational forces into account.

Then, in the case $h_\varphi = 0$, we shall have

$$p_\perp = (P + \rho_0\chi)\mu^{-4}, \tag{7}$$

where $\chi(\xi)$ is an arbitrary function; the remaining functions depend on r and t according to formulas (3), (3′).

In the case $h_z = 0$ we obtain:

$$p_\parallel = (a_1 - \chi'/\xi)T'\mu^{-4}, \tag{8}$$

where $\chi(\xi)$ is an arbitrary function, and the functions $\rho(r,t)$, $p_\perp(r,t)$, $h_\varphi(r,t)$, $\mu(t)$ are determined by formulas (4).

For $h_\varphi \neq 0$, $h_z \neq 0$, and $v_\varphi \neq 0$, solution (5) is valid, in which the constants α and A are arbitrary, while the function $\chi(\xi)$ is found from the formula

$$\chi' = k^2\left[\left(c_5 - \frac{c_1}{k^2}\right)\xi - 4\pi G\frac{R}{\xi} - \frac{2\Phi(\xi)}{\xi^2 R'}\right].$$

Solutions I—III can be used in some specific problems.

The behavior of the functions $f_1(\mu)$, $f_2(\mu)$, which determine various types of gas motion, has been investigated in detail in works (^{3},^{4} ,^{6}). From these

It follows from the investigations that in case I a complete expansion of the plasma in the presence of gravitational forces and a magnetic field is impossible. If $b_3 < 0$, then collapse of the plasma toward the axis of symmetry cannot occur. In case II expansion is possible if $a_5 < 0$, and impossible if $a_5 > 0$. If $a_1 > 0$, then the plasma particles cannot contract into an infinitely thin filament.

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