A POINT EXPLOSION IN A DETONATING GAS

HYDROMECHANICS

1967

SovietRxiv

Abstract

Full Text

UDC 534.222.2

HYDROMECHANICS

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A POINT EXPLOSION IN A DETONATING GAS

(Presented by Academician L. I. Sedov on 2 II 1967)

1. Suppose that in an unbounded, quiescent, perfect gas capable of detonating there has occurred a plane, cylindrical, or spherical point explosion. As usual ([1]), we shall assume that the explosion was produced by an instantaneous release of energy E_0 along a plane ($\nu = 1$), along a straight line ($\nu = 2$), or at a point ($\nu = 3$), with in the plane and cylindrical cases the energy E_0 being calculated, respectively, per unit area or per unit length. As a consequence of the explosion, a strong shock wave will begin to propagate through the gas, heating it to a state in which a rapid combustion reaction is possible. We shall assume that the substance burns in the immediate neighborhood of the shock front, and regard the shock wave together with the zone of chemical reactions behind it as a single surface of strong discontinuity with heat release, i.e., we shall consider a detonation wave.

Let us consider the general picture of the gas flow. It follows from physical considerations that in the first moments after the explosion the medium will move according to the laws of a point explosion, for the contribution of the combustion energy to the total energy balance will still be small. One can estimate those distances up to which it is expedient not to take into account the influence of the detonation energy. Let Q be the heat-producing capacity of unit mass of the combustible medium; let the initial density of the medium ρ_1 and the energy Q be constant. The energy released in the combustion of the medium when the radius of the detonation wave is equal to r_2 has the magnitude

$$\nu^{-1}\sigma_\nu \rho_1 Q r_2^\nu \quad (\sigma_\nu = 2(\nu-1)\pi + (\nu-2)(\nu-3)).$$

Assuming that the explosion energy E_0 is greater than this total detonation energy, we find the condition for a weak influence of the detonation energy on the flow:

$$r_2 < r^*, \qquad r^* = (\nu E_0/\sigma_\nu \rho_1 Q)^{1/\nu}.$$

If the explosion is not a point explosion and the energy E_0 is released in a volume of finite size with characteristic length r_0, then we must also bear in mind that, for the applicability of the conclusions of the theory of a point explosion, the distances r_2 must be several times greater than the length r_0. The indicated conditions restrict the range of application of the laws of an ordinary point explosion to the case of a detonating medium. However, if the energy E_0 is large and is released in a small volume, then for $r < r^*$ the flow will proceed essentially as in a point explosion. On the other hand, for times when $r_2 > r^*$, detonation will play the principal role, and the gas flow will have the main features of ordinary detonation (see on this ([1], [2])). For values of r_2 close to r^*, the gas flow will have a more complicated intermediate character; moreover, the propagation of the detonation wave over large distances may be accompanied by the appearance of secondary shock waves in the disturbed region. One of the distinctive features of the flow under consideration is the presence of a certain region in the neighborhood of the center of explosion in which, by analogy with what occurs in an ordinary point explosion, the gas particles will possess high entropy.

In view of the features of the flow noted above, in the initial stage of a detonation explosion the Chapman–Jouguet rule will not be obeyed. Let us consider this question in somewhat more detail for the case of a gas with constant ratio

of the ratio of specific heats γ. Let c denote the velocity of the shock wave, v_2 the velocity of the gas behind its front, and a_2 the corresponding speed of sound. Then, from the conditions at the shock wave for a strong explosion, we have

$$v_2 + a_2 = \frac{c}{\gamma + 1}\left(2 + \sqrt{2\gamma(\gamma - 1)}\right).$$

It follows from this relation that for $\gamma > 1$ we have $v_2 + a_2 > c$, i.e., at the initial stage of the process we shall have the case of overdriven detonation. As the shock wave propagates in the detonating gas, the quantity $v_2 + a_2$ decreases, and in principle a transition to the Chapman–Jouguet detonation regime is possible here, since the velocity of the shock wave must approach the velocity of steady detonation as $r_2 > r^*$. However, without a complete solution of the problem it is difficult to carry out a detailed analysis of this question. It should be noted that for an explosion in a medium with variable density the Chapman –Jouguet condition may in some cases be satisfied throughout the flow in a point explosion in a detonating gas. Examples of such flows were considered by I. S. Shikin ([3]).

From the mathematical point of view, the solution of the problem under consideration reduces to integrating the system of gas-dynamic equations for one-dimensional motions, taking into account the boundary conditions at the detonation-wave front and the condition that the velocity at the center of symmetry be zero. In addition, it should be taken into account that at the instant $t = 0$ a finite energy E_0 is released at the center, and the conditions are prescribed

$$v(r,0)=0, \qquad \rho(r,0)=\rho_1, \qquad p(r,0)=p_1, \qquad r_2(0)=0.$$

To solve the problem formulated above, numerical methods may be used; for example, a method analogous to that proposed in ([4]) for calculating a point explosion with counterpressure.

2. To obtain a solution with allowance for the detonation energy, one may use the method of linearizing the problem at small times t (for $r_2 < r^*$), i.e., seek the additional perturbations of the flow in the explosion caused by the release of energy at the wave front.

The system of determining parameters of the problem contains the constants $Q, \rho_1, p_1, E_0, \gamma$, and, in addition, the solution will depend on the coordinate r and the time t. For simplicity we shall regard the detonation wave as strong and neglect the initial pressure of the gas p_1. Introduce the effective detonation pressure p_* by the formula

$$p_* = Q\rho_1/\gamma.$$

The quantity p_* may be included in the system of determining parameters instead of Q.

It follows from the system of determining parameters of the problem that, if we introduce the dimensionless functions

$$f=\frac{v}{c}, \qquad g=\frac{\rho}{\rho_1}, \qquad h=\frac{q}{p_*}p, \tag{1}$$

then they will depend on two dimensionless variables, which we take to be

$$\lambda = r/r_2, \qquad q = Q/c^2, \tag{2}$$

and on some constant parameters.

We also introduce the dimensionless radius of the shock wave $R(q)=r_2/r_0$, $r_0=(E_0/p_*)^{1/\nu}$.

In the variables (1), (2), the system of gas-dynamic equations can be written as follows:

$$Df+\frac{1}{\gamma g}\frac{\partial h}{\partial \lambda}-\eta\frac{f}{2q}=0, \qquad Dg+g\left(\frac{\partial f}{\partial \lambda}+(\nu-1)\frac{f}{\lambda}\right)=0, \tag{3}$$

$$Dh+\gamma h\left(\frac{\partial f}{\partial \lambda}+(\nu-1)\frac{f}{\lambda}\right)-\frac{\eta h}{q}=0,$$

where

$$\eta = R\,dq/dR, \qquad D = (f-\lambda)\partial/\partial\lambda + \eta\,\partial/\partial q.$$

From the conditions at the front of the strong detonation wave and at the center of symmetry we find

$$(1-f_2)g_2 = 1, \qquad h_2 = \gamma f_2, \qquad \frac{1}{2}+q = \frac{1}{\gamma-1}\frac{h}{g}+\frac{1}{2}(f_2-1)^2, \qquad f(0,q)=0. \tag{4}$$

Let us note that for $q = 0$ the problem under consideration becomes the problem of a strong explosion in a gas, the exact solution of which was given by L. I. Sedov ([5]).

For the initial stage of the explosion, when $(p_2 - p_*)/p_* > 1$, $q < 0.5(\gamma^2-1)^{-1}$, one can use a way of solving the problem based on linearizing the original equations (3), (4) with respect to the parameter q about the self-similar solution for a strong explosion. The solution of the linearized problem in the adopted variables can be constructed by analogy with the case in which counterpressure is taken into account (see, for example, ([1], [6–8])). We represent the unknown functions in the form

$$\begin{aligned} f(\lambda,q) &= f_0(\lambda) + qf_1(\lambda) + O(q^2),\\ g(\lambda,q) &= g_0(\lambda) + qg_1(\lambda) + O(q^2),\\ h(\lambda,q) &= h_0(\lambda) + qh_1(\lambda) + O(q^2),\\ \frac{R}{q}\frac{dq}{dR} &= \frac{\nu}{1+A_1q+O(q^2)}. \end{aligned} \tag{5}$$

Here $O(q^2)$ denotes terms of order q^2 for small q. If the solution (5) is substituted into system (3) and terms of order q^2 and higher are neglected, then for the linear additions f_1, g_1, h_1 we obtain a system of linear ordinary differential equations. The coefficients of this system will depend on the self-similar solution f_0, g_0, h_0. From this system the functions f_1, g_1, h_1 must be determined. In addition, it is necessary to find the constant A_1, which determines the non-self-similar correction to the law of motion of the shock wave.

From conditions (4) we find the boundary values for the functions f_1, g_1, h_1

$$f_1(1) = -(\gamma-1), \qquad h_1(1) = -\gamma(\gamma-1),$$

$$g_1(1) = -(\gamma+1)^2/f_1(0) = 0. \tag{6}$$

Fig. 1

Figure 1: Fig. 1

In what follows we shall follow work ([7]), where the linearized problem with allowance for counterpressure was considered. It is easy to see that, by virtue of the dimensionless variables introduced above, the system of linear equations for f_1, g_1, h_1 coincides with the corresponding system in work ([7]) (see formulas (1.4) of work ([7])). This system should be solved subject to conditions (6). In the case of arbitrary γ and ν the problem can be solved numerically. By the method developed in ([7]), a calculation was carried out for a number of values of γ at $\eta = 1, 2, 3$. Thus, for $\gamma = 3$ it was found that: $A_1 = 12.3$ ($\nu = 1$), $A_1 = 11.1$ ($\nu = 2$), $A_1 = 10.6$ ($\nu = 3$).

Using the values of A_1, the conditions on the wave (4), and formulas (1.14) of work ([7]), one can determine the law of motion of the detonation wave for small q, and also the change in all parameters at the wave front as a function of the change in dimensionless time.

In Fig. 1, for values $\lambda \geq 0.5$, curves of $p/\beta p_*$ are constructed for $\nu = 3$ for the case $\gamma = 1.2$, $q = 0.15$, $\beta = 1$ (approximate calculation), and curves of $p/\beta p_*$ for the case $\gamma = 3$, $q = 0.04$, $\beta = 5$, the dashed lines corresponding to the explosion without allowance for detonation ($h = h_0$). For $0 \leq \lambda < 0.5$ the pressure changes weakly.

Fig. 1

Let us also note that for $\gamma = 3$, $\gamma = 7$ the linearized problem has an exact analytic solution, by analogy with how this occurs in the problem taking counterpressure into account ([6]).

3. The problem considered above can be generalized to the case of other ideal compressible media different from a perfect gas. This is simplest to do for media for which the expression for the internal energy can be written in the form $\varepsilon = p\varphi(\rho)$, where φ is some function of the density. Self-similar problems of a strong explosion for these media have been studied in ([9]).

One can also consider the case of a variable initial density of the medium $\rho_1 = \rho_1(r)$ or a variable detonation energy $Q = Q(r)$. Here, too, linearized solutions can be constructed by analogy with the problems taking counterpressure into account ([6]).

The author expresses his gratitude to E. Bishimov for assistance in carrying out the calculations.

Mathematical Institute named after V. A. Steklov
Academy of Sciences of the USSR

Received
23 I 1967

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Note: Figure translations are in progress. See original paper for figures.