ON TURBULENT EXCHANGE OVER THE OCEANS

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Abstract

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The turbulent regime of the lower layer of air over the oceans (and also over strongly moistened areas of land) is noticeably affected by the humidity of the air (which we shall characterize by the density of water vapor $\rho\ = \rho c$, where ρ is the density of moist air and c is the specific humidity).

First, owing to the relation

$$\rho'/\rho \approx -T'/T - (R\ /R\ - 1)c', \tag{1}$$

which follows from the equation of state $p = [R\ + (R\ - R\ )c]\rho T$ (where p and T are pressure and temperature, $R\ = 0.287$ and $R\ = 0.461$ J/g · deg are the gas constants of dry air and water vapor; primes here and below denote turbulent fluctuations), humidity affects the magnitude of the buoyancy forces $-g\rho'$, so that their mean work (per unit mass) turns out to be equal to

$$-\frac{g}{\rho}\overline{\rho' w'} = \frac{g}{c_p \rho T}(q_{\text{turb}} + b q_{\text{lat}}); \qquad b = \left(\frac{R}{R} - 1\right)\frac{c_p T}{\mathcal{L}}, \tag{2}$$

where g is the acceleration of gravity; w is the vertical velocity; $c_p \approx 1$ J/g · deg is the specific heat of air at constant pressure; $\mathcal{L} = 2500$ J/g is the latent heat of evaporation; $q_{\text{turb}} = c_p \rho \overline{T' w'}$ is the vertical turbulent heat flux (the overbar here and below denotes averaging); $q_{\text{lat}} = \mathcal{L} \rho \overline{c' w'}$ is the vertical turbulent flux of latent heat ($E = q_{\text{lat}}/\mathcal{L}$ is the rate of evaporation). The coefficient b in (2) is approximately equal to 0.07, but over the oceans q_{lat} is often an order of magnitude larger than q_{turb}, and then the terms in parentheses in (2) are comparable. For a discussion of this humidity effect, see the work of S. S. Zilitinkevich ([1]).

Second, at high air humidity it intensely absorbs long-wave radiation, so that the radiative influx of heat cannot be neglected, as is often done when describing turbulence over land, and the equation of heat influx, neglecting the action of molecular forces, must be written in the form

$$\rho T\, ds/dt = -\partial q_{\text{rad}}/\partial z. \tag{3}$$

Here s is the specific entropy of moist air, z is height, and q_{rad} is the vertical radiative heat flux, which, when only long-wave radiation is taken into account and its scattering is neglected, can be expressed through the known transmittance function of diffuse radiation $D[am]$ in the following way ([2]):

$$q_{\text{rad}}(z) = -\frac{4\sigma T^3}{\pi} \int D \left[a \left| \int_{z_1}^{z} \rho\,(z_2)\, dz_2 \right| \right] \frac{\partial T(z_1)}{\partial z_1}\, dz_1, \tag{4}$$

where $\sigma = 0.817 \cdot 10^{-10}$ cal/cm^2 min $\cdot$ deg^4 is the Stefan-Boltzmann constant; a is the effective absorption coefficient per unit mass (having-

which apparently has a value of order 200 cm^2/g); integration over z_1 is over the whole layer in which the radiation propagates. Here we take into account absorption of radiation only by water vapor; allowing for the additional absorption by carbon dioxide would present no fundamental difficulties.

Third, owing to the relation

$$s' \approx c_p T'/T + (s_{\text{p}} - s_{\text{v}})c', \tag{5}$$

which follows from the definition of the entropy of the mixture $s = (1-c)s_{\text{v}} + cs_{\text{p}}$ (where $s_{\text{v}} = c_{p\text{v}} \ln T - R_{\text{v}} \ln p_{\text{v}} + \text{const}$ and $s_{\text{p}} = c_{p\text{p}} \ln T - R_{\text{p}} \ln p_{\text{p}} + \text{const}$ are the specific entropies of dry air and water vapor, $c_{p\text{v}} \approx c_p,\ c_{p\text{p}} = 1.81$ J/g $\cdot$ deg), the turbulent entropy flux in moist air entering the left-hand side of equation (3) is not proportional to the turbulent heat flux, as in dry air, but also contains the turbulent flux of moisture. In particular,

$$\overline{\rho s w} \approx \rho \overline{s' w'} = \frac{1}{T} \left(q_{\text{turb}} + \frac{s_{\text{p}} - s_{\text{v}}}{c_p} \frac{c_{pT}}{\mathcal{L}} q_{\text{skr}} \right). \tag{6}$$

The terms on the right-hand side of (6), as also in (2), over the oceans are, generally speaking, comparable with one another. This moisture effect has apparently been overlooked until now.

Let us take into account the statistical homogeneity of the ocean surface and consider the stationary regime. Then, beginning at some small height above the wave crests, within a layer in which the action of the Coriolis force is small and there are no evaporating droplets or condensation of moisture, the vertical turbulent fluxes of momentum $-\rho\overline{u'w'}$ and of moisture $\rho\overline{c'w'}$ will be constant with height, i.e.

$$-\overline{u'w'} = u_*^2 = \text{const}; \qquad \rho\overline{c'w'} = \frac{q_{\text{skr}}}{\mathcal{L}} = \text{const} \tag{7}$$

(here u' denotes velocity fluctuations along the mean wind direction, and u_* is the so-called friction velocity). Under these conditions the left-hand side of equation (3) has the form $\rho T\,\overline{ds}/dt \approx T\,\partial\overline{\rho s w}/\partial z$, and the equation itself reduces to $T\overline{\rho s w} + q_{\rm rad} = {\rm const}$, or, taking (6) and the second of equalities (7) into account, to

$$q_{\rm turb} + q_{\rm rad} = q = {\rm const}. \tag{8}$$

The condition $q = 0$, together with stationarity, as explained in (3), corresponds to thermal equilibrium in the atmosphere (from (2) it is seen that the latter, generally speaking, does not coincide with hydrostatic equilibrium, determined by the stratification of density and not of temperature). To equations (7)–(8) we add also the equation of turbulent energy, which, taking (2) into account, assumes the form

$$u_*^2\frac{\partial u}{\partial z} + \frac{g}{c_p\rho T}\left(q_{\rm turb} + b q_{\rm skr}\right) = \varepsilon, \tag{9}$$

where ε is the rate of dissipation of turbulent energy (per unit mass).

For simplicity we shall further restrict ourselves to a description of radiative transfer in the diffusion approximation, when, owing to the large magnitude of α (strong absorption), the factor D under the integral sign in (4) varies much more rapidly than $\partial T/\partial z_1$, so that this quasi-constant second factor (with its value at $z_1 = z$) may be taken outside the integral sign. Then (4) takes the form

$$q_{\rm rad} = -c_p\rho K_{\rm rad}\frac{\partial T}{\partial z}; \qquad K_{\rm rad} = \frac{4\sigma T^3}{\pi c_p\rho}\int D\left[\frac{1}{L_{\rm rad}}\left|\int_{z_1}^{z}\frac{c(z_2)}{c_0}\,dz_2\right|\right]dz_1, \tag{10}$$

where $K_{\rm rad}$ has the meaning of a coefficient of radiative thermal diffusivity, c_0 is an effective value of c (say, the value of c at the lower boundary of the reduced air layer $z = z_0$), and $L_{\rm rad} = 1/\alpha\rho c_0$ is the effective mean free path—

of long-wave photons. Let us also introduce in the usual way the coefficients of turbulent exchange for momentum $K = u_*^2\Big/\dfrac{du}{dz}$, heat $K_T = a_{TK} = -\dfrac{q_{\rm turb}}{c_p\rho}\Big/\dfrac{dT}{dz}$, and water vapor $K_c = a_{cK} = -\dfrac{q_{\rm lat}}{\mathcal{L}\rho}\Big/\dfrac{dc}{dz}$, and then the turbulence scale $l = (K^3/\varepsilon)^{1/4}$.

We pose the problem of determining, with the aid of equations (7)–(10), for given right-hand sides of (7)–(8), the profiles of velocity, temperature, and humidity. In doing so it will be convenient to measure heights by the scale L, and the vertical changes of velocity, temperature, and humidity by the scales $u_*/\varkappa$, T_*, and c_*, where

$$L = -\frac{c_p \rho T u_*^3}{\varkappa g q}; \qquad T_* = -\frac{q}{\varkappa c_p \rho u_*}; \qquad c_* = -\frac{q_{\text{lat}}}{\varkappa \mathcal{L} \rho u_*}. \tag{11}$$

($\varkappa \approx 0.4$ is Kármán' s constant.) We shall use the dimensionless height $\zeta = z/L$ and characterize the profiles by the dimensionless functions

$$\varphi(\zeta) = \frac{\varkappa z}{u_*}\frac{\partial u}{\partial z}; \qquad \varphi_T(\zeta) = \frac{z}{T_*}\frac{\partial T}{\partial z}; \qquad \varphi_c(\zeta) = \frac{z}{c_*}\frac{\partial c}{\partial z}. \tag{12}$$

In these variables equation (9) is reduced to the form

$$\varphi^4 - \zeta\varphi^3 \left(\frac{1}{1 + q_{\text{rad}}/q_{\text{turb}}} + b\frac{q_{\text{lat}}}{q} \right) - \frac{1}{\lambda^4} = 0, \tag{13}$$

where $\lambda = l/\varkappa z$, and the ratio $q_{\text{rad}}/q_{\text{turb}} = K_{\text{rad}}/K_T$ is determined by the following formula, obtained from (10):

$$\frac{q_{\text{rad}}}{q_{\text{turb}}} = \frac{M\varphi}{a_T \zeta} \int \left[D \left| \frac{L}{L_{\text{rad}}} \right| \left| \zeta - \zeta_1 + \frac{c_*}{c_0} \int_{\zeta_1}^{\zeta} d\zeta_2 \int_{\zeta_1}^{\zeta_2} \frac{\varphi(\zeta_3)}{a_c \zeta_3}\, d\zeta_3 \right| \right] d\zeta_1. \tag{14}$$

Here $\zeta_0 = z_0/L$, and M is a dimensionless parameter determined by the formula

$$M = 4\sigma T^3/\varkappa c c_p \rho u_* \tag{15}$$

and, for $T = 300°$, has the value $M \approx 0.4/u_*$, where u_* is measured in cm/sec. If λ, a_T, and a_c are specified with the aid of additional considerations, then from (13)–(14) one can in principle determine φ and $q_{\text{rad}}/q_{\text{turb}}$, after which we also obtain $\varphi_c = \varphi/a_c$ and $\varphi_T = \dfrac{\varphi/a_T}{1 + q_{\text{rad}}/q_{\text{turb}}}$. All these quantities will, generally speaking, be functions of ζ and of the parameters M, $|L|/L_{\text{rad}}$, c_*/c_0, and ζ_0.

In the particular case of hydrostatic equilibrium, when the work of buoyancy forces (2) is equal to zero, we have $q_{\text{turb}} = -bq_{\text{lat}} = \text{const}$, and therefore, according to (8), $q_{\text{rad}} = \text{const}$. Equation (13), under the assumption $\lambda = \text{const}$, leads to a logarithmic law for the wind profile. With $a_T = \text{const}$ and $a_c = \text{const}$, the same law is also obtained for the profiles of temperature and humidity (whereas in dry air for temperature it is obtained only when $q_{\text{turb}} = 0$ and has the pronounced form of an isotherm). This case, apparently, is often observed in nature.

Thermal equilibrium ($q = 0$) proves possible only under isothermy ($\partial T/\partial z = 0$), so that in it $q_{\text{turb}} = q_{\text{rad}} = 0$. In this case, replacing q by q_{lat} in formula (11) for L, the velocity profile φ can be determined from the equation obtained from (13) by replacing the expression in parentheses by the number b. This case is entirely analogous to turbulence in stratified dry air, except that the density stratification will now be determined by the humidity profile and not by temperature. Isothermy is observed over the oceans fairly often.

In the case of convection in the absence of wind ($q > 0,\ u_* = 0$), one may think that strong mixing should lead to an equalization of humidity with increasing height, so that at great heights the atmosphere should become optically homogeneous. At such heights, assuming $c(z) \sim c_0$, for the integral in formula (10) we obtain the approximate value NL_{rad}, where N is a number (in the case of a "gray" homogeneous atmosphere $D(\tau) = 2\pi E_3(\tau)$ and $N = 4\pi/3$), so that $K_l \sim$ const. Since K_T here increases with height, we obtain $q_{\text{rad}}/q_{\text{turb}} \to 0$, i.e., the role of radiative heat exchange vanishes with increasing height, and the asymptotic laws of free convection turn out to have the same form as in dry air (with the only difference being in formula (2) for the work of the Archimedean forces).

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CITED LITERATURE

[1] S. S. Zilitinkevich, *Izv. AN SSSR, Fizika atmosfery i okeana*, No. 10 (1966).
[2] E. M. Feigelson, *Izv. AN SSSR, ser. geofiz.*, No. 10 (1964).
[3] A. S. Monin, *Izv. AN SSSR, Fizika atmosfery i okeana*, No. 6 (1965).

Note: Figure translations are in progress. See original paper for figures.