ON THE MAGNETIC HYDRODYNAMICS OF THE EARTH’ S EXOSPHERE

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Abstract

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Yu. L. MOSKVIN, D. A. FRANK-KAMENETSKII

ON THE MAGNETIC HYDRODYNAMICS OF THE EARTH' S EXOSPHERE

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The precision measurements of the Earth' s magnetic field carried out in recent years have provided extensive information on the morphology of geomagnetic micropulsations ([1]). For the purposes of interpreting these data, we shall consider in greater detail than was done previously the propagation and generation of magnetohydrodynamic (Alfvén and magnetoacoustic) waves in the magnetosphere. For a steady magnetic field with axial symmetry it is convenient to take, as a curvilinear coordinate system, the scalar magnetic potential ψ, the vector magnetic potential, which in this case has only the azimuthal component A_φ, and the azimuthal angle φ, i.e., $x_1 = \psi,\ x_2 = A_\varphi,\ x_3 = \varphi$. The line element is then expressed in the form

$$(ds)^2 = l_1^2(dx_1)^2 + l_2^2(dx_2)^2 + l_3^2(dx_3)^2, \tag{1}$$

where the Lamé coefficients are the quantities

$$l_1 = 1/H, \qquad l_2 = 1/H\rho, \qquad l_3 = \rho; \tag{2}$$

H is the strength of the steady magnetic field, and ρ is the cylindrical radius.

The equations of magnetohydrodynamics for an ideally conducting medium without dissipation in the adopted coordinate system are simplified and lead to wave equations of the form

$$\left(\frac{\omega^2}{H^2 v_A^2} - \frac{m^2}{H^2\rho^2}\right) E_\perp + \frac{1}{H\rho}\frac{\partial}{\partial x_1} H^2\rho^2 \frac{\partial}{\partial x_1}\frac{E_\perp}{H\rho} = \frac{im}{H\rho}\frac{\partial}{\partial x_2}\rho E_\varphi, \tag{3}$$

$$-\frac{\omega^2}{H^2 v_A^2} E_\varphi + \rho\frac{\partial}{\partial x_1}\frac{1}{\rho^2}\frac{\partial}{\partial x_1}\rho E_\varphi + \rho\frac{\partial^2}{\partial x_2^2}\rho E_\varphi = im\rho\frac{\partial}{\partial x_2}\frac{E_\perp}{H\rho},$$

$$v_A^2 = H^2/4\pi\mu,$$

where μ is the plasma density, E_φ and $E_\perp$ are the components of the electric field in the azimuthal direction and along the normal to the magnetic field lines, and the solution is written in the form

$$E \sim e^{-i\omega t + im\varphi}.$$

For the value of the azimuthal number $m = 0$ and in the limiting case of its large values ($m \to \infty$), the system of equations (3) splits into two independent equations: for Alfvén (toroidal) and magnetoacoustic (poloidal) waves. Previously, only the case $m = 0$ had been considered in detail ([2]), whereas both the latest measurements ([3]) and theoretical considerations regarding the mechanism of generation, which we shall now set forth, speak in favor of large values of the azimuthal number. We shall confine ourselves to consideration of this simple limiting case. From (3) we obtain, for Alfvén oscillations,

$$-\frac{\omega^2}{v_A^2 H^2} E_\varphi + \rho \frac{\partial}{\partial x_1} \frac{1}{\rho^2} \frac{\partial}{\partial x_1} \rho E_\varphi = 0. \tag{4}$$

For the case of a dipole magnetic field, we transform (4) to the form

$$\frac{d^2 V}{d\tau^2} + \lambda^2 N(\tau)(1-\tau^2)^6 V + 3\frac{1-6\tau^2}{(1+3\tau^2)^2} V = 0, \tag{5}$$

where $V = (1+3\tau^2)^{-1/2} E_\varphi$; $\tau = \cos\theta$; θ is the polar angle; $\lambda = \omega R_l / v_A(\pi/2)$; $N(\theta) = \mu(\theta)/\mu(\pi/2)$; R_l is the radius of the most distant point of the given field line. The boundary condition at the surface of the Earth is written in Leontovich form ([4])

$$\mathbf{E}_t = \zeta[\mathbf{H}_t \mathbf{n}], \tag{6}$$

where ζ is the effective surface impedance of the ionosphere. For high latitudes condition (6) takes the form

$$V = i\zeta \frac{c}{2\omega R_l} \frac{\partial V}{\partial \tau}, \qquad \tau = 1. \tag{7}$$

To find the fundamental frequency of Alfvén waves we use the Ritz method, assuming in the first approximation for high latitudes that $V = 0$ at $\tau^2 = 1$. Substituting functions of the form

$$V_n = (1-\tau^2)\{a_0 + a_1\tau^2 + \ldots + a_n\tau^{2n}\}$$

into the variational integral, we find:

$$\begin{array}{ll} \text{a)} \quad N(\tau) \equiv 1, \qquad \lambda_1 = 1.58, \qquad V = (1-\tau^2)(1-0.1\tau^2), \\ \text{b)} \quad N(\tau) \equiv H(\tau)/H(\pi/2), \qquad \lambda_1 = 1.29, \qquad V = (1-\tau^2)(1-0.5\tau^2). \end{array} \tag{8}$$

For higher overtones the eigenfrequencies can be found by the quasiclassical method. To determine the behavior of the quasiclassical solution near the boundary, we expand the coefficients of equation (5) in a series in powers of $(1 \pm \tau)$. We obtain

$$\lambda_n = 2.1(n + 0.7); \tag{9}$$

$$V = \left[\frac{(z_0 - z)}{f}\right]^{1/2} J_{1/5}(\lambda(z_0 - z)), \qquad z \geq 0, \tag{10}$$

$$V = \left[\frac{(z_0 + z)}{f}\right]^{1/2} J_{1/5}(\lambda(z_0 + z)), \qquad z \leq 0,$$

where $f(\tau) = (1 - \tau^2)^3 N(\tau)^{1/2}$, $z(\tau) = \int_0^\tau f(\tau)\, d\tau$, $z_0 = z(1) \approx 0.7$.

The imaginary part of the eigenfrequencies, and consequently also the quality factor of the magnetospheric resonator for Alfvén oscillations, is found by the perturbation method, using boundary condition (7). For the fundamental mode we obtain

$$Q_1 \approx 1.2 \left(\operatorname{Re} \frac{c}{2\omega R_l} \zeta\right)^{-1} \tag{11}$$

and for higher overtones

$$Q_n \approx 1.6(n + 0.7)^{8/5} \left(\operatorname{Re} \frac{c}{2\omega R_l} \zeta\right)^{-1}. \tag{12}$$

In the literature ([2],[5]) the mechanism of wave generation due to the instability of a tangential discontinuity during the flow of the solar wind around the magnetosphere has already been discussed. In this case, however, only surface waves are excited, which cannot be detected at the surface of the Earth. Moreover, according to Syrovatskii' s theory (cited in ([4])), the instability increment increases without bound with the azimuthal wave number, producing short-wavelength noise. The mechanism limiting the azimuthal number may be the smearing of the tangential discontinuity due to turbulence in the transition zone. For a smeared discontinuity with velocity $\mathbf{V}$, varying linearly over a width a, we obtain in the most dangerous case $(ku_A) = 0$ dis-

dispersion equation

$$x^2 - x + (e^{-2k_\varphi a} - 1 + 2k_\varphi a)/(2k_\varphi a)^2 = 0, \tag{13}$$

where $x = \omega/k_\varphi v_\varphi,\ k_\varphi = m/R_r$.

It follows from this that the instability disappears for

$$m \gtrsim 1.3R_r/a.$$

According to Explorer 10 satellite data, the width of the tangential discontinuity is $a \sim 3000$ km, whence, for a boundary radius $R_r \sim 60\,000$ km, the greatest azimuthal number of growing perturbations is $m \sim 20$–30, which agrees with the results of measurements ([3]).

Two mechanisms can single out from the surface noise a discrete spectrum of resonant frequencies and ensure their propagation to the Earth' s surface. The first mechanism of transformation is the linear interaction of surface waves with Alfvén waves in a curvilinear magnetic field. Passing in the solution (3) to the next approximation in powers of m^{-1}, we obtain cross terms coupling the Alfvén oscillations with the magnetosonic and surface ones. The relation between the spectral density of magnetic-field fluctuations at the outer boundary of the magnetosphere $S_r(\omega)$ and at the Earth' s surface $S_3(\omega)$ is obtained in the form:

$$S_3(\omega) \approx \sum_n S_r(\omega_n)\, 0.4m \left(\frac{R_\ell}{nR_3}\right)^3 \left(\frac{R_\ell}{R_r}\right)^{2m} \frac{Q_n^2}{1 + Q_n^2(\Delta\omega_n/\omega_n)^2}. \tag{14}$$

Here n is the number of the harmonic of the Alfvén oscillations; $\Delta\omega_n = \omega - \operatorname{Re}\omega_n$.

The factor $(R_\ell/R_r)^{2m}$ should lead to a rapid decrease of the wave amplitude with distance from the auroral zone. In addition, at the same time the frequency increases because of the decrease of R_ℓ and the increase of the Alfvén frequency. Along with such a latitudinal dependence, in a number of cases micropulsations of appreciable amplitude are also observed at middle latitudes and even in the equatorial region. The penetration of waves into these latitudes can be explained either by their propagation along the ionospheric waveguide (under conditions when the phase velocity in the ionosphere is less than $\omega R_3/m$), or by the second, nonlinear mechanism of transformation of surface waves into magnetosonic waves with frequency doubling, analogously to the excitation of microseisms by storms in the ocean ([6]).

Further development of the views set forth requires calculation of the effective surface impedance of the ionosphere and the construction of a nonlinear theory of surface waves. The smallest observed periods of micropulsations, and in particular the carrier frequency of modulated oscillations of the "pearl" type, must already be explained by kinetic instabilities.

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Moscow Institute of Physics and Technology

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Note: Figure translations are in progress. See original paper for figures.