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Abstract

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GEOPHYSICS

B. L. GAVRILIN, A. S. MONIN

A MODEL OF LONG-TERM INTERACTIONS BETWEEN THE OCEAN AND THE ATMOSPHERE

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Following E. N. Blinova ([1,2]) and the work ([3]), we assume that the most important feature of long-term atmospheric processes is their nonadiabatic character. The basic dynamic equation for these processes will be the equation of evolution of the potential vorticity ([4]). In our model, using the quasistatic and quasigeostrophic approximation and taking as constants the Coriolis parameter l and the baroclinicity parameter

$$a^2 = -\frac{T}{T_0}\frac{\partial(S/c_p)}{\partial \ln p}$$

(T and T_0 are the air temperature and its mean near-surface value; p is pressure; S is the entropy of dry air; c_p is its heat capacity at constant pressure), we write this equation in the form

$$\frac{d}{dt}\left(a^2 L^2 \Delta z + \frac{\partial}{\partial p} p^2 \frac{\partial z}{\partial p}\right) = \frac{l}{g} a^2 L^2 \Omega_f - \frac{R}{g}\frac{\partial(p\varepsilon/c_p)}{\partial p}, \qquad (1)$$

where z is the height of isobaric surfaces; t is time; $L = c/l$ is the typical horizontal scale of synoptic processes; $c = \sqrt{RT_0}$ is the speed of sound; R is the gas constant of air; g is the acceleration of gravity; d/dt and Δ are the individual derivative with respect to horizontal motions and the horizontal Laplacian; Ω_f is the vertical component of the vorticity of the frictional forces; $\varepsilon = T(dS/dt + w^*\partial S/\partial p)$ is the density of the heat influx; w^* is the rate of change of pressure in moving air particles.

The vorticity of the frictional forces Ω_f and the heat influx due to turbulent mixing ε_T may be written in the form $\Omega_f = D\Delta\psi$, $\varepsilon_T = c_{pDT}$, where $D = K_h\Delta + \frac{1}{\tau_v}\frac{\partial}{\partial p}p^2\frac{\partial}{\partial p}$ (K_h and $1/\tau_v$ are the coefficients of horizontal and vertical mixing, assumed constant); $\psi = zg/l$ is the stream function of geostrophic horizontal motions, while the temperature T is related to z by the static equation

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$T = -\frac{g}{R}\frac{\partial z}{\partial \ln p}$. Using these expressions, in the left-hand side of (1) one may replace d/dt by $d/dt - D$, and then in the right-hand side it remains to take into account only the radiative and condensation heat influxes $\varepsilon_R + \varepsilon_\phi$. In our model we shall carry out a linearization with respect to the basic state of rest with some "equilibrium" temperature distribution $\overline{T}(p)$, taking into account only horizontal mixing. After such a linearization of equation (1), the operator $d/dt - D$ in its left-hand side is replaced by $\partial/\partial t - K_h\Delta$; anomalies of the radiative heat influx in a cloudless atmosphere will be described by Newton' s law $\varepsilon'_R = -(c_p/\tau_R)T'$, where τ_R is the "time of radiational equalization of temperature inhomogeneities" ; anomalies of the condensation heat influx in clouds, where $\varepsilon'_R = 0$, will be written in the form

$$\varepsilon'_\phi = c_p(1 - a^2/a_B^2)(\partial T'/\partial t - K_h\Delta T'),$$

where a_B^2 is the baroclinicity parameter in a moist-adiabatic atmosphere (henceforth assumed constant).

Assuming that the probability of the presence of clouds in the basic state is equal to 1/2, let us consider a model with mean anomalies of the heat influx ${}^1/_2\varepsilon'_R + {}^1/_2\varepsilon'_\phi$. In it, the linearized equation (1) takes the form

$$\left(\frac{\partial}{\partial t} - K_h\Delta\right)\left(a_0^2L^2\Delta z + \frac{\partial}{\partial p}p^2\frac{\partial z}{\partial p}\right) = -\frac{1}{\tau_0}\frac{\partial}{\partial p}p^2\frac{\partial z}{\partial p}, \tag{1'}$$

where

$$\frac{1}{a_0^2} = \frac{1}{2}\left(\frac{1}{a^2} + \frac{1}{a_B^2}\right) \quad \text{and} \quad \frac{1}{\tau_0} = \frac{1}{2}\frac{a^2}{a_0^2}\frac{1}{\tau_R}.$$

Let us now formulate the boundary conditions at the ocean surface $p = p_0$. One of them will be the requirement that the vertical velocity vanish, which is reduced to the form $w^* = \dfrac{gp_0}{c^2}\dfrac{\partial z}{\partial t}$, or, after determining w^* from the heat-influx equation, to the form

$$\frac{\partial}{\partial t}\left(p\frac{\partial z}{\partial p} + a^2 z\right) = -\frac{R}{g}\frac{\varepsilon}{c_p}. \tag{2}$$

For our model one should put here $\varepsilon = {}^1/_2\varepsilon'_R + {}^1/_2\varepsilon'_\phi$, the only difference from the preceding exposition being that, at the boundary, Newton' s law for radiative heat exchange has the form $\varepsilon'_R = -\dfrac{1}{2}\dfrac{c_p}{\tau_R}(T' - T'_w)$, where T'_w is the water temperature. Then condition (2) is reduced to the form

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$$\frac{\partial}{\partial t}\left(p\frac{\partial z}{\partial p}+a_0^2 z\right)=-\frac{1}{2\tau_0}\left(p\frac{\partial z}{\partial p}+\frac{R}{g}T'_w\right). \tag{2'}$$

The second boundary condition will be that the algebraic sum of all vertical heat fluxes at the ocean surface be equal to zero. Of these we shall take into account, first, the turbulent heat flux in the ocean $c_w\rho_{wK}w\partial T_w/\partial z$, where c_w, ρ_w, and K_w are the specific heat, density, and mixing coefficient in water.

Second, let us consider the turbulent flux of latent heat produced by the evaporation of moisture from the water surface, which has the form $-L(\rho K_z\partial q/\partial z)_{z=0}$, where L is the latent heat of evaporation; ρ, q, and K_z are the density, specific humidity, and coefficient of vertical mixing in air. Taking into account that at the water surface $z=0$ the air is saturated with moisture, i.e.

$$q=q_m=\frac{R}{R_w}\frac{e_m(T)}{p},$$

where R_w is the gas constant of water vapor, and, somewhat underestimating the rate of evaporation in our model, we replace $(\partial q/\partial z)_{z=0}$ by

$$\frac{\partial q_m}{\partial z}=-\frac{\rho g}{p}\left(\frac{T}{e_m}\frac{\partial e_m}{\partial T}\frac{p}{T}\frac{\partial T}{\partial p}-1\right)q_m.$$

Then the anomalies of the evaporation rate will be proportional to $q'_m\approx\dfrac{\partial q_m}{\partial T}T'_w$. We shall write them in the form $c_p\rho\dfrac{H}{\tau_\phi}T'_w$, where

$$\frac{1}{\tau_\phi}=\frac{L}{c_p}\frac{K_z}{H^2}\left(\frac{T}{e_m}\frac{\partial e_m}{\partial T}\frac{p}{T}\frac{\partial T}{\partial p}-1\right)\frac{\partial q_m}{\partial T},$$

with τ_ϕ being the time of vertical transfer of latent heat, and $H=c^2/g$ the so-called height of a homogeneous atmosphere.

Third, let us take into account the flux of long-wave radiation $\rho H\varepsilon_R$, where ε_R is defined in (2).

Fourth, let us consider the flux of short-wave solar radiation $-(1-n)c_p\rho(H/\tau_s)T_0$, where τ_s is the time of arrival of solar heat, and n is the amount of cloudiness. We shall assume the anomalies of this heat flux to be proportional to the anomalies of the amount of cloudiness n' and write them in the form $c_p\rho(H/\tau_s)T_0n'$. For n' we take the empirical formula

$$n'=\frac{1}{w_0^*}(\rho g w)_{p=p_1},$$

where $p_1\approx 850$ mb is the level of maximum recurrence–

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cloud cover, and according to Lewis' data ([5]), $w_0^* \approx 0.9$ mb/hour. Here we shall assume

$$\rho g w = \rho g \frac{\partial z}{\partial t} - w^* \approx \frac{gp}{a^2 c^2}\left[\frac{\partial}{\partial t}\left(p\frac{\partial z}{\partial p} + \alpha^2 z\right) + \frac{R}{g}\frac{\varepsilon'}{c_p}\right],$$

where $\varepsilon' = c_p K_h \Delta T' + \frac{1}{2}(\varepsilon'_R + \varepsilon'_\phi)$, as in (1′). As a result the second boundary condition takes the form

$$\left[\frac{R}{g}\left(\frac{c_w\rho_w}{c_p\rho}\frac{K_w}{H}\frac{\partial T'_w}{\partial z} + \frac{T'_w}{\tau_\phi}\right) + \frac{1}{2\tau_R}\left(p\frac{\partial z}{\partial p} + \frac{R}{g}T'_w\right)\right]_{p=p_0} = -\frac{p_1}{\tau_s w_0^* a_0^2}\left[\left(\frac{\partial}{\partial t} + \frac{1}{\tau_0} - K_h\Delta\right)p\frac{\partial z}{\partial p} + \alpha_0^2\frac{\partial z}{\partial t}\right]_{p=p_1}. \tag{3}$$

In our model we shall not take into account currents and horizontal mixing in the ocean, and for the anomalies of water temperature T'_w we use the ordinary heat-conduction equation $\partial T'_w/\partial t = K_w \partial^2 T'_w/\partial z^2$ with the condition of damping at some depth $z = -H_w$. Further, we shall study elementary wave solutions of the equations for $z(x, y, p, t)$ and $T'_w(x, y, z, t)$, depending on x, y, t according to the law $\exp(i(m_1 x + m_2 y)/L - i\omega t)$. Then for $z = 0$ one has

$$\frac{c_w\rho_w}{c_p\rho}\frac{K_w}{H}\frac{\partial T'_w}{\partial z} = \delta\,\frac{T'_w}{\tau_w}\sqrt{i\tau_w\omega}\,\mathrm{cth}\,\sqrt{i\tau_w\omega}, \tag{4}$$

where $\delta = c_w\rho_w H_w/c_p\rho H$, $\tau_w = H_w^2/K_w$ is the mixing time of the ocean. The mixing coefficient in the ocean is usually small as a result of the strongly stable density stratification; thus, for example, calculations by R. V. Ozmidov and N. I. Popov ([6]) for the vertical distribution of Sr^{90} in the Atlantic Ocean gave the estimate $K_w = 30$ cm^2/sec. In this case, even for a small thickness of the mixing layer $H_w \sim 500$ m, the time τ_w is large, of the order of several years. Below we shall consider oscillations with considerably smaller periods, i.e. $\omega \gg 1/\tau_w$, and it will be possible to assume cth $\sqrt{i\tau_w\omega} \approx 1$.

Next, the solution of equation (1′) for z, possessing the required regularity as $p \to 0$, will depend on p according to the law p^μ, where

$$\mu = -\frac{1}{2} + \sqrt{\frac{1}{4} + \alpha_0^2 m^2\left(1 - \frac{1}{1 + (\tau_0/\tau_h)m^2 - i\tau_0\omega}\right)}, \tag{5}$$

where $m = \sqrt{m_1^2 + m_2^2}$ is the dimensionless horizontal wave number, and $\tau_h = L^2/K_h$ is the time of horizontal mixing of the atmosphere, whose magnitude is apparently of the order of weeks. According to our estimates, $\tau_0 \sim 10^7$ sec; in this case, for oscillations with periods much greater than $\frac{(2\pi)^2}{m^2}\tau_h$, the dependence of μ on ω may be neglected.

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For the dimensionless frequency of oscillations of the ocean–atmosphere system $X = \tau_0\omega(1+\alpha_0^2/\mu)$, in our model, from the boundary conditions (2′) and (3), taking into account the simplifications indicated above, the following secular equation is obtained:

$$(1-2iX)(\sqrt{iX}-\xi)=\eta, \tag{6}$$

where the following notation has been adopted:

$$\xi=\lambda\left(\zeta-\frac{\tau_0}{\tau_\phi}-\frac{\alpha^2}{\alpha_0^2}\right);\qquad \eta=\lambda\left[\zeta\left(1+2\frac{\tau}{\tau_h}m^2\right)+\frac{\alpha^2}{\alpha_0^2}\right];$$

$$\lambda=\frac{1}{\delta}\sqrt{\frac{\tau_w}{\tau_0}\left(1+\frac{\alpha_0^2}{\mu}\right)};\qquad \zeta=\frac{1}{2}\frac{p_1(p_1/p_0)^\mu}{\tau_s w_0^* a_0^2}. \tag{7}$$

The most interesting case appears to be $2\frac{\tau_0}{\tau_h}m^2 \gg 1$, in which η is large and equation (6) is approximately reduced to the form $-2iX\sqrt{iX}=\eta$, whence $X \approx {}^1/_2(\eta/2)^{2/3}(\pm\sqrt{3}+i)$, which corresponds to growing oscillations with period

$$\frac{2\pi}{\omega}\approx\frac{4\pi}{\sqrt{3}}\left[2\delta\,\frac{\tau_s w_0^* a_0^2}{p_1(p_1/p_0)^\mu}\,\frac{\tau_h}{\sqrt{\tau_0\tau_w}}\left(1+\frac{a_0^2}{\mu}\right)\right]^{2/3} m^{-4/3}\tau_0, \tag{8}$$

varying, for waves of length L (i.e., for the case $m=2\pi$), depending on the value of τ_h ($\sim$ 10-100 days), within the limits of 1-5 months (in addition, for large η there is also a less interesting solution $X=-i(\eta/2)^{2/3}$, corresponding to monotonically decaying standing waves). As the wavelength increases their period increases, but for very long waves the accuracy of equation (6) ceases to be sufficient, and the simplifications used must be abandoned; i.e., in (6) one must introduce, at $\sqrt{iX}$, the factor cth $\sqrt{i\tau_w\omega}$ and take into account the dependence of μ on ω.

The model considered, owing to the simplifications adopted in it (above all, the possibility of avoiding an explicit description of the humidity field and of obtaining linear equations), of course does not claim to provide a quantitative description of the large-scale processes of interaction between the ocean and the atmosphere, but it does make it possible to draw the qualitative conclusion that these interactions can lead to the generation of oscillations with periods of the order of months. This conclusion should be of interest for the physical theory of long-range weather forecasts.

Institute of Oceanology
Academy of Sciences of the USSR

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Note: Figure translations are in progress. See original paper for figures.

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