# RADIATIVE EFFECTS IN EXPERIMENTS WITH COLLIDING ELECTRON BEAMS

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## Abstract

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# RADIATIVE EFFECTS IN EXPERIMENTS WITH COLLIDING ELECTRON BEAMS

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**1.** Recently the first experimental results have been obtained on the scattering of high-energy electrons in colliding beams ([1,2]). As the accuracy of the experiment increases—which is necessary for testing the applicability of quantum electrodynamics at small distances (and this is the main purpose of experiments with colliding electron beams)—in comparing experiment with theory it will be necessary to take into account radiative corrections to the electron–electron scattering cross section. As is known, the radiative corrections contain contributions both from virtual photons (and pairs) and from the emission of real photons. This is connected with the fact that, in the scattering of charged particles through a nonzero angle, accelerations always arise, leading to photon emission and appearing in the structure of the theory in the form of an infrared divergence. The total cross section of the elastic process (with vacuum corrections included) and of the inelastic process (with emission of real photons) in the given order in $e^2$ contains no infrared divergence, but, naturally, depends essentially on the experimental conditions. For this reason the study of photon-emission processes in electron collisions is of considerable interest. It should be noted that both soft and hard photons are emitted mainly into narrow cones in the directions of motion of the initial and final particles (if one restricts oneself to logarithmic accuracy, then only these cones need be taken into account).

In the present work simple expressions are found for the cross sections of the process of electron scattering with emission of a photon of arbitrary energy into narrow cones along the directions of motion of the particles. Using these expressions, general formulas are obtained for the radiative corrections to the electron–electron scattering cross section. Finally, the cross section for emission of soft photons is calculated for arbitrary electron scattering angles.

**2.** Let us consider the process of emission of a photon with arbitrary energy along the momentum $p_1$ ($p_1, p_2$ are the momenta of the initial particles; $p_3, p_4$ are the momenta of the final particles; $k$ is the photon momentum), when the angular dimensions of the detector satisfy $2\vartheta_0 \ll 1$, in the case of large electron scattering angles $\theta \gg 1/\gamma$ ($\gamma = E/m$). To obtain the cross section of this process we shall use formulas (2.1)–(2.5) of paper ([4]). Since the integration over the photon emission angle will be carried out with accuracy up to terms of order $\vartheta_0^2, 1/\gamma^2$, in all quantities entering these formulas, except $(kp_1)$, one may put $\vartheta_k = 0$ ($\vartheta_k$ is the angle between the vectors $k$ and $p_1$). Selecting the leading terms, we obtain, with the indicated accuracy*:

$$d\sigma_1 = \frac{\alpha^3}{8\pi^2}\frac{d\xi\, d\Omega\, d\Omega_k}{(1-c^2)^2}\left[f^2(\xi) + \frac{(1+c)^4 + (1-c)^4(1-\xi)^4}{f^2(\xi)}\right]\times$$

$$\times\left[\frac{1}{(kp_1)}\left(1+\frac{1}{(1-\xi)^2}\right) - \frac{m^2\xi}{(1-\xi)(kp_1)^2}\right], \tag{1}$$

* Here and below the final electrons are assumed to be ultrarelativistic, i.e. $1 - \xi \gg 1/\gamma$.

where

$$\xi = \omega/E, \qquad c = \cos\theta, \qquad f(\xi) = 2 - \xi(1-c). \tag{2}$$

From formula (1) it is clear that, for soft photons, the cross section has a maximum at $\vartheta_k \sim 1/\gamma$, decreasing at $\vartheta_k = 0$ by approximately a factor $\gamma^2$. Since terms of order $1/\gamma^2$ were discarded, formula (1) gives an incorrect picture of the dependence of the cross section on $\vartheta_k$ for $\vartheta_k \ll 1/\gamma$ in the case of soft photons. This, however, does not affect the applicability of the integral cross section, owing to the small contribution of these angles. Such a decrease of the soft-photon emission cross section at very small angles leads to a relative broadening of the peak in the cross section for soft photons in comparison with the peak for hard photons.

After trivial integration over the photon-emission angles, we obtain*

$$d\sigma_1 = \frac{r_0^2\alpha}{4\pi\gamma^2}\frac{d\xi}{\xi}\frac{d\Omega}{(1-c^2)^2}\left[f^2(\xi) + \frac{(1+c)^4 + (1-c)^4(1-\xi)^4}{f^2(\xi)}\right]\times$$

$$\times\left[\left(1+\frac{1}{(1-\xi)^2}\right)\ln(1+n^2) - \frac{2n^2}{(1+n^2)(1-\xi)}\right], \tag{3}$$

where $n = \vartheta_0\gamma$.

The cross section for radiation along the momentum $\mathbf{p}_2$ has the same form, if the angle is measured from the direction of $\mathbf{p}_2$. Just as simply one can obtain the cross section for emission of a photon along the momentum of the final particle $d\sigma_3$. This cross section is found to be equal to

$$d\sigma_3 = d\sigma_0\,\frac{\alpha}{4\pi^2}(1-\xi)\, d\xi\, d\Omega_k\, E^2\left[\frac{1+(1-\xi)^2}{(kp_3)} - \frac{m^2\xi}{(kp_3)^2}\right], \tag{4}$$

where $d\sigma_0$ is the Møller cross section.

Integrating over the photon-emission angle, we obtain

$$d\sigma_3 = d\sigma_0 \frac{\alpha}{2\pi} \frac{d\xi}{\xi} \left[ (1 + (1-\xi)^2) \ln (1 + n^2(1-\xi)^2) - \frac{2n^2(1-\xi)^3}{1 + n^2(1-\xi)^2} \right]. \quad (5)$$

It is characteristic that the cross sections (4), (5) depend on the scattering angle as the Møller one does, whereas the angular dependence in formulas (2), (3) goes over into the Møller dependence only as $\xi \to 0$. This reflects the fact that emission of hard photons along the direction of motion of the initial particles distorts the angular distribution of the final particles, whereas for emission in the direction of the final momenta no such distortion occurs.

3. Let us proceed to consideration of the large-angle scattering cross section in the case when the emitted photons are not registered. We shall be interested in the radiative corrections to this cross section. At present, in experiment two electrons flying apart after scattering are recorded, but their energies are not measured. Thus, by a "scattering event" one understands all cases in which photon emission does not lead to a significant noncollinearity of the momenta of the final electrons. As was already noted, the cause of noncollinearity is the emission of hard photons in the direction of motion of the initial particles; therefore the admissible angle of noncollinearity is related to the limiting energy of photons emitted in this direction. On the other hand, emission of photons in the directions of motion of the final particles does not lead to noncollinearity, and hence no restrictions on their energy arise.

The vacuum contributions to the radiative corrections and the contributions of soft-photon emission have been calculated repeatedly (see, for example, ([3,4])). Therefore

* It should be noted that if one of the electrons is scattered through an angle $\theta$, then, for a given photon energy $\xi$, the second electron emerges at an angle $\chi$, with the accepted accuracy,

$$\cos\chi = 1 - \frac{2(1+c)}{[2 - \xi(2-\xi)(1-c)]}.$$

one should consider the contribution from the emission of hard photons. For this purpose we shall use the cross sections obtained in the preceding section. If one takes into account that, in the limit $\omega \to 0$, these formulas go over into the corresponding expressions with classical currents, then it is clear that we can write down the cross section for electron–electron scattering through a large angle, with radiative corrections allowing for the emission of photons of arbitrary frequency. For the reason indicated above, we shall take as different the limiting frequencies of photons emitted along the directions of motion of the initial particles $\xi_1, \xi_2$ and the directions of motion of the final particles $\xi_3 = \xi_4$. To logarithmic accuracy* this cross section has the form

$$d\sigma = d\sigma_0 \left\{ 1 + \frac{22}{3}\frac{\alpha}{\pi}\ln\gamma + \frac{4\alpha}{\pi}\ln\gamma\left[\ln\xi_3 - \xi_3 + \frac{\xi_3^2}{4}\right] \right\} + \\ + \frac{r_0^2 d\Omega}{\gamma^2}\frac{\alpha}{\pi}\ln\gamma \left\{ \frac{1}{(1-c)^2}\left[\ln\frac{\xi_1^2}{1-\xi_1} + \frac{\xi_1}{1-\xi_1} + \ln\left(\frac{f(\xi_1)}{r}\right) + \frac{2c}{f(\xi_1)} - c\right] + \right. \\ + \frac{1}{(1+c)^2}\left[2\ln\xi_1 - 2\xi_1 + \frac{\xi_1^2}{r}\right] + \frac{1}{2}\ln\frac{2\xi_1}{f(\xi_1)} + \frac{(1-c)\xi_1}{2f(\xi_1)} + \\ \left. + \text{terms } (c \to -c,\ \xi_1 \to \xi_2) \right\}. \tag{6}$$

Let us note here that in previous works on the calculation of radiative corrections to the cross section for electron–electron scattering in colliding beams ([3,4]), under the assumption that the final electrons are registered by pairs of counters, an averaging over the counters was carried out. Expression (6) contains radiative corrections to the "scattering events."

4. Let us consider the emission of soft photons ($\xi \ll 1$), but for electron scattering through an arbitrary angle ($\theta > 1/\gamma$). This question is of interest, since scattering through small angles can be used for monitoring the fact of beam collision. In this connection the question arises of the radiation accompanying such scattering. Since only electrons that have lost a small part of their energy are registered, it is precisely the emission of soft photons that is of interest. We shall again be interested in the emission of photons into a small angle $\vartheta_0 \ll 1$ along the direction of motion of the initial electrons, and the calculation will again be carried out to an accuracy up to terms $\vartheta_0^2$, $1/\gamma^2$.

We shall use the known formulas for the emission of soft photons (see, for example, ([5])). A simple analysis shows that, in the present formulation of the problem, the contribution to the differential cross section in $\xi$ is given only by the terms associated with the emission of a photon by the given particle. The contributions from emission by the other particle, as well as the interference terms, may be neglected in this approximation. Then we obtain

$$d\sigma = d\sigma_0\, dW(\omega),$$

$$dW(\omega) = \frac{\alpha}{2\pi}\frac{d\omega}{\omega}\left\{ \frac{2x^2+1}{x\sqrt{1+x^2}} \ln \frac{(1+n^2)(x+\sqrt{1+x^2})^3}{2\left[2x(1+x^2) - \gamma^2 x(1-\beta\mu) + \sqrt{1+x^2}\,\beta\gamma\sqrt{z^2+(1-\mu^2)}\right]} + \frac{z}{\sqrt{z^2+(1-\mu^2)}} - 1 - \frac{2n^2}{1+n^2} \right\}, \tag{7}$$

where

$$4m^2x^2 = (p_1 - p_3)^2 = 4|\mathbf{p}_1|^2 \sin^2\theta/2, \qquad \mu = \cos\vartheta_0,$$

$$z = \gamma(\mu - \beta) + 2x^2/\beta\gamma. \tag{8}$$

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* To this accuracy the contributions from photon emission along the momenta $p_3$ and $p_4$ are the same. It is assumed that the admissible noncollinearity angle $\Delta\vartheta \gg 1/\gamma$.

It is evident that at large scattering angles formula (7) goes over into formula (3), if one assumes that in the latter $\xi \to 0$:

$$d\sigma = d\sigma_0 \frac{\alpha}{\pi}\frac{d\omega}{\omega}\left[\ln(1+n^2) - \frac{n^2}{1+n^2}\right]. \tag{9}$$

It follows from expression (9) that the probability of emission of a soft photon in scattering through large angles does not depend on the scattering angle. This circumstance has a transparent physical meaning and is connected with the fact that, for scattering through large angles, only the initial particle emits into the small angle $\vartheta_0$. For this reason, to logarithmic accuracy, the cross section for the emission of soft photons into a cone is equal to one half of the total cross section for emission by the given particle (see [5], formula (9)).

It is important to note that the arguments presented above show that measuring the cross section of a process with photon emission means measuring the corresponding contribution to the radiative corrections. This circumstance can be used for a direct measurement of these contributions.

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*Note: Figure translations are in progress. See original paper for figures.*